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In 2008, worldwide equity markets collapsed and many assets which conventional investment wisdom until then regarded as effective equity diversifiers, such as commodities, also experienced dramatic falls. Meanwhile, equity volatility skyrocketed, causing long positions in equity volatility to rally. These events, as well as regulatory developments, dashed the exaggerated hopes placed in traditional forms of diversification and led investors to pay increased attention to the volatility and downside risk of equity holdings, if not to question the level of their allocation to equity altogether. They also prompted interest in the possible use of equity volatility derivatives as diversifiers for traditional and alternative portfolios in general, and equity positions in particular.

Against this backdrop, the present publication is dedicated to exploring the uses of volatility derivatives by professional investors, with specific emphasis on their equity portfolio management applications.

The research shows how volatility derivatives can be used to optimise access to the equity risk premium in a controlled volatility-risk environment, and to engineer equity portfolios with attractive downside-risk properties.

The results we obtain suggest that a long volatility position shows a strongly negative correlation with respect to the underlying equity portfolio and that adding a long volatility exposure to an equity portfolio would result in a substantial improvement of the risk-adjusted performance of the portfolio. The benefits of the long volatility exposure are found to be the strongest in market downturns, where they are needed the most.

The benefits of adding volatility exposure to equity portfolios are also found to be robust with respect to the introduction of trading costs associated with rolling over volatility derivatives contracts.

We hope that you will find the results of this research both informative and useful.

We would like to express our sincere gratitude to our longstanding partners at Eurex for supporting this research.

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Executive Summary
1. Introduction

Recent market turbulence, coupled with the presence of increasingly strict regulatory constraints have led institutional investors (pension funds, insurance companies) and asset managers to monitor the volatility and downside risk of their equity holdings with increased scrutiny. One approach towards the design of equity portfolios in the presence of tight risk budgets involves building equity portfolio benchmarks with the lowest possible volatility. Over the past few years, this approach has gained considerable popularity in the industry and a large number of asset management firms are currently offering global minimum variance (GMV) portfolios.

Whether investing in a GMV portfolio is the most efficient and robust route for managing equity volatility remains, however, an open question. From an academic perspective, this approach is not consistent with standard portfolio theory, which instead suggests first identifying the maximum Sharpe ratio (MSR) portfolio, as opposed to the GMV portfolio, and then mixing that portfolio with cash so as to achieve the target volatility consistent with investors’ risk appetites and budgets. In other words, while the GMV is an efficient portfolio in the absence of a risk-free asset, it is no longer an efficient portfolio when a risk-free asset is introduced.

In this article, we analyse a competing approach to the design of attractive equity solutions with managed volatility, based on mixing well-diversified maximum Sharpe ratio portfolios with volatility derivatives. Intuitively, one expects that a portfolio strategy mixing a well-diversified equity benchmark and a suitably designed long exposure to volatility through trading in volatility index futures and/or volatility index options can be engineered so as to provide access to the equity risk premium while allowing for an explicit management of the volatility risk budget.

A number of studies suggest that volatility and equity returns tend to move in opposite directions (i.e. they are strongly negatively correlated) which allows for significant diversification benefits from adding a long volatility position to equity portfolios. In addition, the negative correlation between an implied volatility and underlying equity portfolio is found to be strongest in large market downturns. One possible explanation for the negative correlation of equity volatility to equity market is the "leverage effect" (Black 1976; Christie 1982; Schwert 1989): a decrease (respectively, an increase) in equity prices increases (respectively, decreases) the company’s leverage, thereby increasing (respectively, decreasing) the risk to equity holders and increasing (respectively, decreasing) equity volatility.

Another alternative explanation (French et al. 1987; Bekaert et Wu 2000; Wu 2001; Kim et al. 2004) is the "volatility feedback effect": assuming that volatility is incorporated in stock prices, a positive volatility shock increases the future required return on equity and stock prices are expected to fall simultaneously. The presence of profound economic reasons that explain the inverse relationship between equity return and volatility is a comforting indication of the robustness of the diversification benefits to be expected. It stands in contrast with the well-known lack of robustness of portfolio diversification within the equity universe (e.g. international diversification), where
The Benefits of Volatility Derivatives in Equity Portfolio Management — May 2012

Executive Summary

Diversification is known to fail precisely when it is needed the most because of the convergence of all correlations to one in periods of high market turbulence.

The focus of this paper is to provide a formal analysis of the benefits of volatility derivatives in equity portfolio management from the perspective of a European investor. Our main contribution is to compare the risk/return characteristic of equity portfolios combined with long volatility exposure to those of a GMV equity portfolio – the conventional approach to managing equity volatility. Our paper is in fact the first to provide an explicit comparison of managed volatility strategies based on GMV portfolios and managed volatility strategies based on volatility derivatives. Our results unambiguously suggest that the latter approach is a more efficient way to manage equity volatility, especially in market downturns periods.

Our main results can be summarised as follows. Using European data, we first confirm that the correlation between the return on volatility indexes and the return on equity indexes is strongly negative, with an absolute level of correlation that increases in recessions and/or high volatility regimes compared to the unconditional estimates. We then show that even a relatively modest allocation to volatility derivatives, consistent with a reasonable level of expected performance, can allow an investor to generate equity portfolios that have more attractive downside risk properties compared to GMV portfolios, with a substantial reduction in maximum drawdown levels. These findings are robust with respect to the introduction of trading costs associated with rolling over volatility futures contracts, so as to generate the target level of long volatility exposure. We also analyse the benefits of adding volatility option positions, and found substantial benefits over the sample period, even though the sample size is too limited because of data availability for the results to be fully informative.

2. Long-term Analysis Conducted at Index Level
The VSTOXX index, based on EURO STOXX 50 real-time options, is designed to reflect the market expectations of equity price volatility. By definition, the VSTOXX index is a measure of an expected volatility in the market which is expected to be strongly negatively correlated with EURO STOXX 50 series. As plotted in Figure 1, both series indeed seem to move in opposite directions. To get a first sense of the relationship, we first estimated an unconditional correlation between the VSTOXX and EURO STOXX 50 index return series based on the full sample period ranging from January 1999 to April 2011. Based on the analysis, the results confirm a substantial negative correlation of -0.74 (-0.73 and -0.66, respectively, for weekly and monthly data) between the two series for the sample period.

In further analysis, we checked whether the results are robust with respect to changes in time period and market conditions. In order to prove that the relationship is consistent over time, we generated 5-year rolling window correlation estimates between the two index series returns on the period ranging from January 1999 to April 2011. Based on the analysis, the results confirm a substantial negative correlation of -0.74 (-0.73 and -0.66, respectively, for weekly and monthly data) between the two series for the sample period.

In further analysis, we checked whether the results are robust with respect to changes in time period and market conditions. In order to prove that the relationship is consistent over time, we generated 5-year rolling window correlation estimates between the two index series returns on the period ranging from January 1999 to April 2011. Based on the analysis, the results confirm a substantial negative correlation of -0.74 (-0.73 and -0.66, respectively, for weekly and monthly data) between the two series for the sample period.

2 - One additional contribution of the paper is to confirm with European data similar results previously obtained with US data.
increasingly so for more recent time periods. We also analysed whether the negative correlation is robust with respect to changes in market conditions. In order to distinguish between the periods of high and low volatility in the European equity market, we used a Markov regime-switching model (Perlin 2010). Using weekly EURO STOXX 50 Index data, we distinguished between the states of high, medium and low volatility. The estimated correlation between EURO STOXX 50 and VSTOXX index returns is -0.76, -0.73 and -0.62 for the periods of high, medium and low volatility, respectively.

In addition, we used the National Bureau of Economic Research (NBER) recession/expansion indicators as a control variable (see Figure 1). The negative correlation seems particularly pronounced during the periods indicated as NBER recessions. During NBER recessions the correlation level reached -0.78, which is relatively close to the correlation level (i.e. -0.76) estimated during high volatility periods as defined with the Markov regime-switching model.

During the recent 2008 crisis, the negative correlation between VSTOXX and EURO STOXX 50 indexes was particularly strong, estimated at -0.80 (the highest value so far) for the January 2008 to December 2008 period. These results suggest that the benefits of diversification with volatility indices manifest themselves when they are needed the most.

Next, we analysed the benefits of adding a long volatility exposure to the equity portfolio. We use the EURO STOXX 50 index to represent a large cap European stock benchmark. In this section, we only simulated a long volatility position by ‘trading’ in the VSTOXX spot index. Although a direct investment in the VSTOXX index is not possible in practice, this ‘theoretical’ approach allowed us to access a longer data history (i.e. January 1999 to April 2011) for analysing portfolios’ performances.

We constructed a number of equity portfolios with 5% increasing allocations to a long volatility exposure. A long volatility...
position itself would hardly generate any positive return – 100% investment in VSTOXX resulted in 0.3% p.a. over the sample period (additionally, 100% investment in VIX had a return of -0.7% p.a.) – consistent with the view in current literature that there is a negative risk premium associated with being long volatility. However, gradually increasing the equity portfolio's allocations to volatility exposure has a very positive effect on equity portfolio performance. We have illustrated this effect in Figure 2, in an efficient frontier format.

The pure equity portfolio is clearly inferior to other investment opportunities, bearing in mind the existence of a diversified portfolio on the efficient frontier that has the same standard deviation as the equity portfolio, but that offers significantly higher returns. In our sample, the maximum Sharpe ratio (0.46) is achieved for the portfolio with 30% allocation to VSTOXX and 70% allocation to EURO STOXX 50.

We then extended the analysis by comparing the performance between the equity portfolio with a long volatility exposure and a GMV portfolio. We used MSCI Europe Minimum Volatility Index as a proxy GMV portfolio in our study. For this analysis, we selected the portfolio with long volatility exposure that had similar volatility to that of a GMV portfolio over the sample period ranging from December 2001 to April 2011.

The results, illustrated in Figure 3, are clearly in favour of the diversified equity portfolio with a long volatility exposure. Both portfolios have relatively similar performances during the periods of low volatility and the diversified portfolio with VSTOXX exposure always outperforms the GMV portfolio in the periods of high volatility. Although volatilities of both portfolios for the sample period are similar, the returns are significantly improved in the diversified portfolio with VSTOXX exposure case (i.e. 9.7% compared to a 2.1% return of GMV portfolio).

Figure 2: Impact of Adding Long Volatility Exposure to Equity Portfolio in 5% Increments

The effects of adding VSTOXX index exposure to EURO STOXX 50 portfolio in 5% increments, estimated based on the sample period ranging from January 1999 to April 2011.

Figure 3: Performance Comparison between Equity Portfolio with Long Volatility Exposure and GMV Portfolio

The results illustrated in Figure 3 clearly show that the diversified equity portfolio with a long volatility exposure outperforms the GMV portfolio. Both portfolios have relatively similar performances during the periods of low volatility, but the diversified portfolio with VSTOXX exposure always outperforms the GMV portfolio in the periods of high volatility. Although volatilities of both portfolios for the sample period are similar, the returns are significantly improved in the diversified portfolio with VSTOXX exposure case (i.e. 9.7% compared to a 2.1% return of GMV portfolio).

Overall, the results in this section provide strong evidence of the benefits of adding a long volatility exposure to an equity portfolio. We also demonstrated that adding a volatility exposure to the equity portfolio not only improves its performance as compared to a pure equity case, but it can also provide a more efficient method of managing downside volatility exposure than the GMV approach.

3. Implementing the Analysis with Volatility Futures

In the previous section a structural volatility exposure was represented by a ‘theoretical’ direct investment in the VSTOXX index. However, in practice, the VSTOXX index is not directly investable, and in order to invest in VSTOXX, an investor may take a position in VSTOXX futures and/or options contracts. Mini-futures on VSTOXX were introduced on the Eurex Exchange in June 2009, with a contract value of €100 per index point. They replaced previously listed futures on VSTOXX, which had a contract size of €1,000 EUR per index point. In our analysis, we used the data of both currently trading and delisted VSTOXX futures series to obtain a longer data history (the total combined sample period ranges from April 2008 to April 2011).

Considering that an individual futures contract is traded for limited time only, an investor has to roll over the initial VSTOXX investment over the series of consecutive futures contracts for a long exposure in VSTOXX. We constructed three separate VIX futures series based on different rollover methodologies: 1-month, 3-month and longest-traded (LT) series. The purpose of this exercise is to analyse the costs associated with different rollover strategies as a function of the frequency of rebalancing. We have estimated that during the analysed sample period from April 2008 to April 2011, the VSTOXX futures market was approximately 79% in contango and 21% in backwardation. As a result, a rollover strategy typically induces a negative return.
Executive Summary

A few recent papers report that the majority of futures contracts have a lower and less variable carry when rolled over 5 days prior to maturity, rather than waiting until maturity. However, this observation is specific to the US market and we found no evidence that rolling over 5 days prior to maturity significantly improved the results in portfolios with long volatility exposure. Considering this result, we used rollover at maturity in all our further empirical analyses.

In order to be consistent with our earlier analysis, we again constructed several equity portfolios with increasing allocations to VSTOXX futures positions. As before, European equity market exposure is approximated by the investment in the EURO STOXX 50 Index. The investment in VSTOXX is represented by a fully collateralised VSTOXX futures position. The analysis starts with the pure equity portfolio as a benchmark case and adds, in 5% increments, a long volatility exposure to the portfolio. The results for the best performing 3-month VSTOXX futures series are presented in Figure 4, in an efficient frontier format. It is interesting to note that the best performing portfolio is achieved by allocating 30% to VSTOXX futures, which is a similar result to that obtained with VSTOXX index data.

In order to access the full impact of transaction costs, we incorporated the bid-ask spread into the analysis. Including the bid-ask spread costs significantly affects the performance of VSTOXX futures – the returns decreased by 26%, 12.6% and 8.5% p.a. for 1-month, 3-month and longest-term (LT) futures, respectively.

We also compared the performance of the diversified portfolio with a managed volatility position with VSTOXX futures to one of the GMV portfolios. The diversified portfolio with VSTOXX futures proves to be a better investment opportunity than the GMV portfolio. Firstly, it improves the standard deviation of returns from 21.6% (for GMV portfolio) to 16.1% p.a.

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4 - Lee and Lin (2010), Alexander and Korovilas (2011)
(for diversified portfolio); secondly, it also reduces negative returns for the sample period – from -2.8% (GMV) to -1.0% p.a. (diversified portfolio). In summary, the results obtained in this section suggest that the benefits of adding a long volatility exposure to equity portfolios are robust with respect to the introduction of trading costs involved in implementation with volatility futures contracts. Careful attention to trade execution is nonetheless required to limit the negative impact of transaction costs, negative carry and roll yield on volatility futures during normal periods.

4. Short-term Analysis with Volatility Options

In this section, we considered a different approach based on the use of volatility options for gaining a long exposure to volatility. March 2010 witnessed the introduction of option contracts on the VSTOXX index, which provided investors with more flexibility for trading European volatility.

In order to assess the impact of adding VSTOXX options to equity portfolios, we constructed a long volatility position by rolling over one month to expiration VSTOXX call options. We use both at-the-money (ATM) and out-of-the-money (10% OTM and 25% OTM) calls for our analysis. Considering that volatility options are much more sensitive to changes in underlying volatility compared to fully collateralised futures contracts, we used 1% increments in volatility exposure rather than 5% increments used for VSTOXX futures.

The performance of ATM VSTOXX calls provides very similar results to the VSTOXX futures. While adding a small positive exposure to the volatility index option portfolio slightly improves (1% and 2%) the performance of the overall portfolio, further increases provide no additional value. Due to increased sensitivity, the results achieved with OTM calls is much more favourable than those achieved with ATM calls; and, in the case of the 25% OTM calls, the return improvements are impressive.
The performance of the portfolios with increasing allocation to 25% OTM calls is depicted in an efficient frontier form in Figure 5.

In further analysis, we estimated the impact of the bid-ask spread on the performance of the diversified portfolios with VSTOXX options exposure. In this case, the drag on performance amounts to 0.53%, 4.28% and 6.21% p.a. for ATM, 10% OTM and 25% OTM calls, respectively.

We also considered a classic strategy for managing downside risk in equity portfolios – the use of protective puts. In every financial textbook, protective puts are referred to as a direct hedge for the price movements in equity portfolios. In order to test this assertion, we compared the performance of an equity portfolio with VSTOXX call allocations to that of an equity portfolio mixed with long EURO STOXX 50 puts. We find that equity portfolios with EURO STOXX 50 put positions do not perform as well as portfolios mixed with VSTOXX calls. None of the portfolios with EURO STOXX 50 puts have better ‘adjusted’ Sharpe ratios than those of a pure equity portfolio.

Up to this point, we have mostly focused on the diversification properties of volatility derivatives. However, an investor can also use VSTOXX options to trade on a specific view on the VSTOXX direction or volatility changes. In this section, we analyse the performance of two commonly used strategies for generating premium: (i) short out-of-the-money VSTOXX puts; and (ii) VSTOXX ratio spread strategy. Both strategies can be used as more innovative ways for equity portfolio management.

However, in both cases, a careful selection of option strike prices proved to be critical for portfolio performance. Therefore, it is important to take current market volatility conditions into account when designing and implementing an option trading strategy.

However, the data history available for VSTOXX options is very short (ranging from March 2010 to April 2011). Due to an extremely short data history and corresponding sample size, it would be difficult to provide a formal analysis of the marginal benefits to be expected from using volatility index futures as opposed to volatility index options. Therefore, the analysis in this section is merely to be regarded as an example of an alternative way of structuring a long volatility exposure.

5. Conclusion

In this paper, we analyse a novel approach in the design of attractive equity solutions with managed volatility, based on mixing a well-diversified equity portfolio with volatility derivatives, as opposed to minimising equity volatility through minimum variance approaches. The results we obtain suggest that a long volatility position shows a strongly negative correlation with respect to the underlying equity portfolio and that adding a long volatility exposure to an equity portfolio would result in a substantial improvement of the risk-adjusted performance of the portfolio. The benefits of the long volatility exposure are found to be strongest in market downturns, when they are most needed.
We also compare the performance of the diversified equity portfolios including volatility derivatives with that of global minimum variance (GMV) portfolios that are commonly used in industry as a benchmark strategy for reducing portfolio risk. We found that the diversified portfolio with long volatility exposure is a more efficient approach for managing risk.

We also consider the challenges related to a practical implementation of this strategy by using derivatives instruments – futures and options – that allow investors direct access to trading volatility. We consider how increasing allocation to volatility derivatives affects the portfolio performance; we also evaluate transaction costs in each case and discuss the advantages and disadvantages of using each type of instrument. The benefits of adding volatility exposure to equity portfolios are found to be robust with respect to the introduction of trading costs associated with rolling over volatility derivatives contracts.
1. Introduction
Recent market turbulence, coupled with the presence of increasingly strict regulatory constraints have led institutional investors (pension funds, insurance companies) and asset managers to monitor the volatility and downside risk of their equity holdings with increased scrutiny. One approach towards the design of equity portfolios in the presence of tight risk budgets involves building equity portfolio benchmarks with the lowest possible volatility. Over the past few years, this approach has gained considerable popularity in the industry, and the non-exhaustive list of firms that are currently managing global minimum variance (GMV) portfolios includes Acadian Asset Management, AXA Rosenberg, Invesco, LGT Capital Management, MSCI Barra, SEI, Robeco, State Street Global Advisors and Unigestion, among others. Currently, GMV portfolios are largely promoted as pragmatically useful benchmarks for investors or asset managers wishing to benefit from some fraction of the equity risk premium without the full associated downside risk.

Whether investing in a GMV portfolio is the most efficient and robust route for managing equity volatility remains, however, an open question. From an academic perspective, this approach is not consistent with standard portfolio theory, which instead suggests first identifying the maximum Sharpe ratio (MSR) portfolio, as opposed to the GMV portfolio, and then mixing that portfolio with cash so as to achieve the target volatility consistent with investors’ risk appetites and budgets. In other words, while the GMV is an efficient portfolio in the absence of a risk-free asset, it is no longer an efficient portfolio when a risk-free asset is introduced.

In this article, we analyse a competing approach to the design of attractive equity solutions with managed volatility, based on mixing well-diversified maximum Sharpe ratio portfolios with volatility derivatives. Intuitively, one expects that a portfolio strategy mixing a well-diversified equity benchmark and a suitably designed long exposure to volatility through trading in volatility index futures and/or volatility index options can be engineered so as to provide an access to the equity risk premium while allowing for an explicit management of the volatility risk budget.

The rapid development of standardised products, especially (exchange-traded) volatility index futures and (OTC-traded) variance swaps, has recently provided investors with straightforward access to a wide range of strategies for gaining structural exposure to volatility. One of main motivations for trading in volatility is precisely to diversify equity risk through long implied volatility exposure. A number of studies\(^5\) suggest that volatility and equity returns tend to move in opposite directions (i.e. they are strongly negatively correlated) which allows for significant diversification benefits from adding a long volatility position to equity portfolios. In addition, the negative correlation between an implied volatility and underlying equity portfolio is found to be strongest in large market downturns. One possible explanation for the negative correlation of equity volatility to equity market is the "leverage effect" (Black 1976; Christie 1982; Schwert 1989): a decrease (respectively,  

1. Introduction

An increase in equity prices increases (respectively, decreases) the company’s leverage, thereby increasing (respectively, decreasing) the risk to equity holders and increasing (respectively, decreasing) equity volatility. Another alternative explanation (French et al. 1987; Bekaert et Wu 2000; Wu 2001; Kim et al. 2004) is the "volatility feedback effect": assuming that volatility is incorporated in stock prices, a positive volatility shock increases the future required return on equity and stock prices are expected to fall simultaneously. The presence of profound economic reasons that explain the inverse relationship between equity return and volatility is a comforting indication of the robustness of the diversification benefits to be expected. It stands in contrast with the well-known lack of robustness of portfolio diversification within the equity universe (e.g. international diversification), where diversification is known to fail precisely when it is needed the most because of the convergence of all correlations to one in periods of high market turbulence.

The risk diversification benefits of long volatility exposure, however, come at a cost. Recent academic research has found that there is a positive risk premium over time to being short volatility or conversely, that there is a negative risk premium to being long volatility. In equilibrium, because of the negative correlation between market index returns and market index volatility, buyers of options may be willing to pay a premium because a long position in volatility helps hedge market-wide risk (Bakshi and Kapadia 2003). In other words, because volatility is negatively correlated with the returns to equities, investors are willing to pay a premium to hold this asset. In a recent paper, Carr and Wu (2009) find that the negative correlation between stock index returns and the return variance generates a strongly negative beta, which would explain a low or even negative expected return on the long volatility exposure. They also find that this negative beta only explains a small portion of the negative variance risk premiums. Other risk factors identified by the recent literature, such as size, book-to-market, and momentum are also unable to explain the strongly negative variance risk premiums, and they conclude that the majority of the market variance risk premium is generated by an independent variance risk factor.

The focus of this paper is to provide a formal analysis of the benefits of volatility derivatives in equity portfolio management from the perspective of a European investor. Our main contribution is to compare the risk/return characteristic of equity portfolios combined with long volatility exposure to those of a GMV equity portfolio – the conventional approach to managing equity volatility. Our paper is in fact the first to provide an explicit comparison of managed volatility strategies based on GMV portfolios and managed volatility strategies based on volatility derivatives. Our results unambiguously suggest that the latter approach is a more efficient way to manage equity volatility, especially in market downturns periods.6

More specifically, our main results can be summarised as follows. Using European data, we first confirm that the correlation between the return on volatility indexes and the return on equity indexes is

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6 - One additional contribution of the paper is to use European data to confirm similar results previously obtained with US data.
1. Introduction

strongly negative, with an absolute level of correlation that increases in recessions and/or high volatility regimes compared to the unconditional estimates. We then show that even a relatively modest allocation to volatility derivatives, consistent with a reasonable level of expected performance, can allow an investor to generate equity portfolios that have more attractive downside risk properties compared to GMV portfolios, with a substantial reduction in maximum drawdown levels. These findings are robust with respect to the introduction of trading costs associated with rolling over volatility futures contracts, so as to generate the target level of long volatility exposure. We also analyse the benefits of adding volatility option positions, and found substantial benefits over the sample period, even though the sample size is too limited because of data availability for the results to be fully informative.

The remainder of the paper is organised as follows. In Section 2, we analyse the relationship between EURO STOXX 50® and VSTOXX® index series, construct a number of portfolios with varying long volatility exposure and compare their performance with that of a GMV. Section 3 provides a practical insight on how this strategy can be implemented with VSTOXX futures. In Section 4, we discuss an alternative approach for gaining long volatility exposure by using VSTOXX derivatives. Finally, Section 5 concludes.
2. Long-term Analysis Conducted at Index Level
2. Long-term Analysis Conducted at Index Level

As discussed in the introduction, a number of academic studies discuss the economic reasons for the existence of a strong negative correlation between the equity volatility and underlying equity markets. In what follows, we will provide empirical confirmation of this negative link through a detailed analysis of the correlation between VSTOXX® and EURO STOXX 50® indexes, and also discuss the portfolio implications of these findings.

2.1. Analysis of the Correlation Between EURO STOXX 50 and VSTOXX indexes

The VSTOXX index is based on EURO STOXX 50 real-time options and is designed to reflect the market expectations of equity price volatility by measuring the square root of the implied variance across all options of a given time to expiration. By definition, the VSTOXX index is a measure of an expected volatility in the market and, therefore, should be strongly negatively correlated with EURO STOXX 50 series (Bakshi and Kapadia 2003; Carr and Wu 2010; Dash and Moran 2007; Alexander and Korovilas 2011). To get a first sense of the relationship, we first estimated an unconditional correlation between VSTOXX and EURO STOXX 50 index return series based on the full sample period ranging from January 1999 to April 2011 (see Figure 1). We used the daily (as well as weekly/monthly) return data for VSTOXX and EURO STOXX 50 indexes available from Datastream. Based on the analysis, the results confirm a substantial negative correlation of -0.74 (-0.73 and -0.66, respectively, for weekly and monthly data) between two series for the sample period.

In further analysis, we checked whether the results are robust with respect to changes in time period and market conditions. In order to prove that the relationship is consistent over time, we generated 5-year rolling window correlation estimates between the two index return series on the period ranging from January 1999 to April 2011. The analysis confirms the correlation level to be systematically negative, irrespective of the time period under consideration and increasingly so for more recent time periods (see Figure 2).

We also analysed whether the negative correlation is robust with respect to changes in market conditions. In order to distinguish between the periods of high and low volatility in the European equity market, we used a Markov regime-switching model (Perlin 2010). Using weekly EURO STOXX 50 Index data, we distinguished between the states of high, medium and low volatility. The estimated correlation between EURO STOXX 50 and VSTOXX index returns is -0.76, -0.73 and -0.62 for the periods of high, medium and low volatility, respectively. This clearly supports the assumption that negative correlation between two series tends to increase with higher volatility in the market.

In addition, we used the National Bureau of Economic Research (NBER) recession/expansion indicators as a control variable (see Figure 3). The negative correlation seems particularly pronounced during the periods indicated as NBER recessions. During NBER recessions the correlation level reached -0.78, which is relatively close to the correlation level (i.e. -0.76) estimated during high volatility periods as defined with the Markov regime-switching model.

2. Long-term Analysis Conducted at Index Level

Figure 1: Time Evolution of EURO STOXX 50 Index and VSTOXX Index

Daily time series for EURO STOXX 50 and VSTOXX indexes on the sample period ranging from January 1999 to April 2011.

Figure 2: Correlation of EURO STOXX 50 Index and VSTOXX Index across Time

EURO STOXX 50 index negative correlation with VSTOXX index, based on 5-year daily return rolling window data on the sample period ranging from January 1999 to April 2011.
2. Long-term Analysis Conducted at Index Level

As an additional robustness check, we closely analysed the correlation between VSTOXX and EURO STOXX 50 index returns during the recent 2008 crisis (see Figure 4). This period is of a particular interest, because it is a perfect example of both equity market prices and their volatility behaviour during a severe market breakdown. Several academic studies (i.e. Szado 2009; Toikka et al. 2004; Lee and Lin 2010; Alexander and Korovilas 2011) suggest that the negative correlation between the equity index and its implied volatility should be the strongest in large downward moves.

Indeed, the analysis confirms that the negative correlation between VSTOXX and EURO STOXX 50 indexes is particularly strong, estimated at -0.80 (the highest value so far) during the January 2008 to December 2008 period. The results suggest that a long position in volatility might be a particularly effective diversifier in major downward market moves such as the recent crisis.

We also have estimated the correlation level in more recent periods starting from January 2010 to April 2011 (see Figure 5). The results show that the negative correlation between the returns of both indexes remained strong at -0.81, despite the fact that the market was much less volatile during this period. This analysis is consistent with the previous results using 5-year rolling window correlation estimates (see Figure 2), referring that the correlation between VSTOXX and EURO STOXX 50 indexes has increased over time and is higher in more recent time periods.

In summary, the correlation level between EURO STOXX 50 index and VSTOXX index series remained negative for all market conditions (i.e. high/low volatility, growth/recession) with a significant increase in major downward market moves and, also, higher in more recent time periods.
2. Long-term Analysis Conducted at Index Level

In this section, we analyse the benefits of adding a long volatility exposure to the equity portfolio. A number of studies\textsuperscript{4} find that the strong negative correlation between an implied volatility and underlying equity portfolio results in significant diversification benefits from adding a long volatility position to the equity portfolio, and we want to assess whether those benefits can also be found based on European data.

\textbf{2.2. Portfolio analysis}

Figure 4: EURO STOXX 50 Index and VSTOXX Index Performance in 2008

Daily time series for EURO STOXX 50 and VSTOXX indexes on the sample period ranging from January 2008 to December 2008.

Figure 5: EURO STOXX 50 Index and VSTOXX Index Performance in 2010/2011

Daily time series for EURO STOXX 50 and VSTOXX indexes on the sample period ranging from January 2010 to April 2011.

2. Long-term Analysis Conducted at Index Level

We use the EURO STOXX 50 index to represent a large cap European stock benchmark. In this section, we only simulate a long position in the VSTOXX index. Although a direct investment in the VSTOXX index is not possible in practice, and while we will focus on investment in VSTOXX derivatives in further sections, this ‘theoretical’ approach allows us to access a longer data history for analysing portfolios’ performances. The sample period ranges from January 1999 to April 2011 and again both VSTOXX and EURO STOXX 50 index levels are obtained from Datastream.

For the purpose of this analysis, we constructed a number of equity portfolios with increasing allocations to a long volatility exposure. The analysis starts with the pure equity portfolio as a benchmark case and adds (in 5% increments) a long volatility exposure to the portfolio. We use a number of traditional parameters, including portfolio returns, volatility, Sharpe ratio, skewness, excess kurtosis, and historical VaR (daily) at 95% threshold, to compare performances of the portfolios. The results are presented in Table 1.

In both cases, the results show that a long volatility position itself would hardly generate any positive return – 100% investment in VSTOXX resulted in 0.3% p.a. over the sample period and 100% investment in VIX had a return of -0.7% p.a. – consistent with the view in current literature that there is a negative risk premium associated with being long volatility (Bakshi and Kapadia 2003; Carr and Wu 2009). However, gradually increasing the portfolio’s allocations to volatility exposure has a very positive effect on equity portfolio performance – it increases total portfolio returns and decreases standard deviations of returns. Then, after a certain point (~25-30% for VSTOXX allocations) further allocation to long volatility exposure starts increasing the overall portfolio volatility as well. We have illustrated this effect in Figure 6, in an efficient frontier format. The pure equity portfolio is clearly inferior to other investment opportunities, bearing in mind the existence of a diversified portfolio on the efficient frontier that has the same standard deviation as all equity portfolios, but that offers significantly higher returns. In our sample, the maximum Sharpe ratio (0.46) is achieved for the portfolio with 30% allocation to VSTOXX and 70% allocation to EURO STOXX 50.

There is a strong empirical evidence that returns on long volatility positions are not normal (Carr and Wu 2009; Hafner and Wallmeier 2008). We therefore extended our investigation to include the impact of skewness and kurtosis. The results indicate that equity portfolio returns are usually negatively skewed (-0.72 on the sample period for EURO STOXX 50), and adding a long volatility exposure has a positive impact on the portfolio skewness. For example, the maximum Sharpe ratio portfolio (i.e. 30% allocation to VSTOXX and 70% to EURO STOXX 50) has the highest positive skewness value as well. On the other hand, adding a long volatility exposure to the portfolio also increased the portfolio kurtosis, which indicates higher probability of obtaining an extreme value in the future. In our sample, introducing an exposure to volatility risk leads to a relatively substantial reduction in the overall portfolio extreme downside risk, estimated in terms of the portfolio historical VaR.
2. Long–term Analysis Conducted at Index Level

Table 1: Summary Statistics for Portfolios with Increasing Allocation to Long Volatility

| Jan/1999 – Apr/2011 | 100% EURO STOXX 50 | 95% EURO STOXX 50 + 5% VSTOXX | 90% EURO STOXX 50 + 10% VSTOXX | 85% EURO STOXX 50 + 15% VSTOXX | 80% EURO STOXX 50 + 20% VSTOXX | 75% EURO STOXX 50 + 25% VSTOXX | 70% EURO STOXX 50 + 30% VSTOXX | 65% EURO STOXX 50 + 35% VSTOXX | 60% EURO STOXX 50 + 40% VSTOXX | 55% EURO STOXX 50 + 45% VSTOXX | 50% EURO STOXX 50 + 50% VSTOXX | 45% EURO STOXX 50 + 55% VSTOXX | 40% EURO STOXX 50 + 60% VSTOXX | 35% EURO STOXX 50 + 65% VSTOXX | 30% EURO STOXX 50 + 70% VSTOXX | 25% EURO STOXX 50 + 75% VSTOXX | 20% EURO STOXX 50 + 80% VSTOXX | 15% EURO STOXX 50 + 85% VSTOXX | 10% EURO STOXX 50 + 90% VSTOXX | 5% EURO STOXX 50 + 95% VSTOXX | 100% VSTOXX |
|---------------------|---------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| Ann. Return         | -1.52%              | 1.40%                       | 4.08%                       | 6.50%                       | 8.64%                       | 10.49%                      | 12.04%                      | 13.27%                      | 14.18%                      | ...                         | 3.00%                       | 0.32%                       |                             |                             |                             |                             |                             |                             |                             |
| Ann. Std Deviation  | 24.41%              | 20.20%                      | 16.73%                      | 14.54%                      | 14.23%                      | 15.01%                      | 19.06%                      | 23.09%                      | 27.61%                      | ...                         | 84.95%                      | 90.33%                      |                             |                             |                             |                             |                             |                             |                             |
| Sharpe Ratio*       | -0.19               | -0.89                       | 0.05                        | 0.23                        | 0.38                        | 0.46                        | 0.46                        | 0.44                        | 0.40                        | ...                         | -0.00                       | -0.03                       |                             |                             |                             |                             |                             |                             |                             |
| Skewness            | -0.72               | -0.45                       | -0.02                       | 0.06                        | 1.23                        | 1.73                        | 1.86                        | 1.63                        | 1.24                        | ...                         | 0.99                        | 1.04                        |                             |                             |                             |                             |                             |                             |                             |
| Excess Kurtosis     | -0.17               | -0.02                       | 0.34                        | 1.12                        | 2.36                        | 3.70                        | 4.34                        | 3.86                        | 2.65                        | ...                         | 0.54                        | 0.65                        |                             |                             |                             |                             |                             |                             |                             |
| Historical VaR (daily) | -2.40%            | -1.94%                      | -1.52%                      | -1.25%                      | -1.23%                      | -1.31%                      | -1.54%                      | -1.84%                      | -2.24%                      | ...                         | -7.07%                      | -7.52%                      |                             |                             |                             |                             |                             |                             |                             |

*Note: The resulted negative Sharpe ratios are included only for the completeness of analysis; in general, negative Sharpe ratio would indicate that risk-free asset performs better than the portfolio being analysed and it is difficult to interpret.

Figure 6: Impact of Adding Long Volatility Exposure to Equity Portfolio in 5% Increments

The effects of adding VSTOXX index exposure to EURO STOXX 50 portfolio by 5% increments, estimated based on the sample period ranging from January 1999 to April 2011.

For comparison purposes, we have repeated the same exercise with US market data for which a longer data history is available (VIX data is available starting January 1990). We used S&P500 data to represent an investment in equity and VIX - a long position in volatility. The results we obtain, shown in Table 2 below, are qualitatively similar to those obtained with European data and on a shorter sample period.
2. Long-term Analysis Conducted at Index Level

The benefits of adding a long volatility exposure can be clearly seen when comparing the performance of diversified portfolios with that of a pure equity portfolio over time (see Figure 7). All diversified portfolios outperformed the EURO STOXX 50 portfolio at each point in time, with the maximum Sharpe ratio portfolio (30% in VSTOXX + 70% in EURO STOXX 50) resulting in significantly higher returns and lower volatility over the sample period.

However, it should be noted that further allocations of more than 30% to VSTOXX may negatively affect Sharpe ratios for the portfolio due to the impact of a negative risk premium on long equity exposure that eventually offsets the benefits of the reduction in risk.

Table 2: Summary Statistics for Portfolios with Increasing Allocation to Long Volatility (US market)

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<td>Ann. Return</td>
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<td>15.07%</td>
<td>16.71%</td>
<td>18.02%</td>
<td>19.01%</td>
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<td>..</td>
<td>2.42%</td>
<td>-0.72%</td>
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<tr>
<td>Ann. Std Deviation</td>
<td>18.20%</td>
<td>14.41%</td>
<td>11.99%</td>
<td>11.81%</td>
<td>13.94%</td>
<td>17.57%</td>
<td>21.97%</td>
<td>26.79%</td>
<td>31.76%</td>
<td>..</td>
<td>90.29%</td>
<td>95.70%</td>
</tr>
<tr>
<td>Sharpe Ratio*</td>
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<td>0.51</td>
<td>0.80</td>
<td>0.98</td>
<td>0.86</td>
<td>0.83</td>
<td>0.71</td>
<td>0.60</td>
<td>0.52</td>
<td>..</td>
<td>0.63</td>
<td>-0.01</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.78</td>
<td>-0.73</td>
<td>-0.59</td>
<td>-0.33</td>
<td>0.03</td>
<td>0.40</td>
<td>0.70</td>
<td>0.89</td>
<td>1.01</td>
<td>..</td>
<td>1.06</td>
<td>1.04</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>0.35</td>
<td>0.14</td>
<td>-0.29</td>
<td>-0.79</td>
<td>-1.03</td>
<td>-0.73</td>
<td>-0.05</td>
<td>0.63</td>
<td>1.09</td>
<td>..</td>
<td>0.56</td>
<td>0.46</td>
</tr>
<tr>
<td>Historical VaR (daily)</td>
<td>-1.74%</td>
<td>-1.29%</td>
<td>-1.02%</td>
<td>-0.98%</td>
<td>-1.16%</td>
<td>-1.47%</td>
<td>-1.85%</td>
<td>-2.27%</td>
<td>-2.69%</td>
<td>..</td>
<td>-7.88%</td>
<td>-8.32%</td>
</tr>
</tbody>
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*Note: The resulted negative Sharpe ratios are included only for the completeness of analysis; in general, a negative Sharpe ratio would indicate that risk-free asset performs better than the portfolio being analysed and as such is difficult to interpret.

Figure 7: Performance of EURO STOXX 50 Index and Portfolios with VSTOXX Exposure

Daily time series for EURO STOXX 50 Index and diversified portfolios with VSTOXX exposure on the sample period ranging from January 1999 to April 2011.
2. Long-term Analysis Conducted at Index Level

2.3. Comparison with a Global Minimum Variance Portfolio

We then extend the analysis by comparing the performance between the equity portfolio with a long volatility exposure and a GMV portfolio. Global minimum variance portfolios are commonly used in the industry as a practical benchmark for an equity portfolio with a managed downside risk exposure. We used MSCI Europe Minimum Volatility Index as a proxy GMV portfolio in our study. The MSCI Europe Minimum Volatility Index is calculated by optimising an underlying MSCI Europe Index (that covers 16 developed countries including Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the United Kingdom) by using an estimated covariance matrix (based on the BARRA model) to produce an index that has the lowest absolute volatility. The daily data on MSCI Europe Minimum Volatility Index performance is available on Bloomberg starting from December 2001.

For this analysis, we selected the portfolio with long volatility exposure that had similar volatility to that of GMV portfolio over the sample period ranging from December 2001 to April 2011. The closest portfolio in term of volatility of returns can be approximated by portfolio with 30% allocation to VSTOXX and 70% allocation to EURO STOXX 50 (with volatility of 18.4% p.a. versus 16.8% p.a. of GMV portfolio).

The results, illustrated in Figure 8, are clearly in favour of the diversified equity portfolio with a long volatility exposure. Both portfolios perform relatively similar during the periods of low volatility and the diversified portfolio with VSTOXX exposure always outperforms the GMV portfolio in the periods of high volatility. Although volatilities of both portfolios for the sample period are similar, the returns are significantly improved in the diversified portfolio with VSTOXX exposure case (i.e. 9.7% compared to a 2.1% return of GMV portfolio). The estimated Sharpe ratio for the diversified portfolio is relatively high.
2. Long-term Analysis Conducted at Index Level

at 0.39 and the Sharpe ratio for the GMV portfolio is very close to 0.

The advantages of the diversified equity portfolio with a long volatility exposure are particularly clear during a recent market crisis of 2008. Figure 9 shows the performances of both GMV and equity portfolios with a long volatility exposure during this period. Holding the diversified portfolio with VSTOXX exposure not only protects portfolio value, but also results in a 10% gain during this period, while the GMV portfolio loses 37.2% of its value. The GMV portfolio also has much higher volatility during this period, reaching to 28.7% p.a. as compared to 22.1% p.a. for the diversified equity portfolio with a long volatility exposure.

We also compared the performances of GMV and equity portfolio with a long volatility exposure during a more recent period starting January 2010 to April 2011 (see Figure 10). Both portfolios follow each other closely during this period where an equity portfolio with long volatility exposure results in slightly higher return of 9.5% p.a., compared to the 6.8% p.a. return of the GMV portfolio. However, the volatility of an equity portfolio with long volatility exposure is also higher at 20.1% p.a., compared to the GMV portfolio’s volatility of 15.3% p.a.

An equity portfolio with a long volatility exposure, again, shows a superior performance in May 2010 down market, when rising concerns regarding sovereign crises were followed by the “Flash Crash of 2:45” on May 6th.

Overall, the results in this section provide strong evidence of the benefits of adding
a long volatility exposure to an equity portfolio. As it appears, even a small addition of allocation to VSTOXX has a potential to improve portfolio efficiency. We also demonstrate that adding a volatility exposure to the equity portfolio not only improves its performance as compared to a pure equity case, but it can also provide a more efficient method of managing downside volatility exposure than the GMV approach.

However, it should be noted that, so far, we have used a ‘hypothetical’ investment in the VSTOXX index. We will focus on a more practical implementation of this strategy through trading in VSTOXX futures contracts in the next section.
2. Long-term Analysis Conducted at Index Level
3. Implementing the Analysis with Volatility Futures
In the previous section a structural volatility exposure was represented by a ‘theoretical’ direct investment in the VSTOXX index. However, in practice, the VSTOXX index is not directly investable, and in order to invest in VSTOXX, an investor may take a position in VSTOXX futures and/or options contracts.

3.1. Constructing Time Series with VSTOXX Futures

Mini-futures on VSTOXX were introduced on the Eurex Exchange in June 2009, with a contract value of €100 per index point. They replaced previously listed futures on VSTOXX, which had a contract size of €1,000 per index point. The data including bid-ask prices and trading volumes on currently traded VSTOXX mini futures as well as delisted VSTOXX futures is obtained from the intraday transaction records provided by Eurex. In our analysis, we used the data of both currently trading and delisted VSTOXX futures series to obtain a longer data history. The total combined sample period ranges from April 2008 to April 2011, which is, of course, substantially smaller than the sample period used in our previous analysis based on volatility index level data.

Considering that an individual futures contract is traded for limited time only, an investor has to roll over the initial VSTOXX investment over the series of consecutive futures contracts for a long exposure in VSTOXX. Following Alexander and Korovilas (2011), we constructed three separate VIX futures series based on different rollover methodologies: 1-month, 3-month and longest-traded (LT) series. All series start with an initial investment in futures contracts on the first day of the sample. On the rollover day the algorithm chooses the next month available contract (in the 1-month series) or the next available contract on the quarterly cycle (the 3-month series). The third series, the longest-traded (LT) series, always rolls over into the longest maturity contract that is actively traded. The purpose of this exercise is to analyse the costs associated to different rollover strategies as a function of the frequency of rebalancing.

Corresponding VSTOXX futures term structure curves can have one of two shapes: in contango, the futures prices for short maturities are less expensive than those maturing later; or, in backwardation, the opposite is true. Figure 11 shows VSTOXX implied volatility term structure on the 19 May 2010, when market was in backwardation, and Figure 12 indicates a contango market observed on the 8 October 2010. The VSTOXX futures market is typically in contango; backwardation is experienced only during a period of unusual high volatility. We have estimated that during the analysed sample period from April 2008 to April 2011, the VSTOXX futures market was approximately 79% in contango and 21% in backwardation. As a result, a rollover strategy typically induces a negative return.

A few recent papers (Lee and Lin 2010; Alexander and Korovilas 2011) report that the majority of futures contracts have a lower and less variable carry when rolled over 5 days prior to maturity, rather than waiting until maturity. However, this observation is specific to the US market and is, mostly, due to the maturity effect,
3. Implementing the Analysis with Volatility Futures

which is exacerbated by the settlement process for VIX futures. The underlying VIX index is based on average bid and ask option prices, but VIX futures are settled on the special opening quotation (SOQ) price. The SOQ is extracted using actual traded prices of SPX options during the market open at settlement day. Consequently, the difference between the VIX futures settlement price and VIX deviated from zero. This convergence problem leads to increased arbitrage trading activity over the last few days prior to maturity, causing increased volatility in VIX futures prices as they approach the last trading day.

Figure 11: VSTOXX Futures Term Structure in Backwardation, the 19th May, 2010

![VSTOXX Futures Term Structure in Backwardation, 19th May, 2010](image)

Figure 12: VSTOXX Futures Term Structure in Contango, 8th October, 2010

![VSTOXX Futures Term Structure in Contango, 8th October, 2010](image)
3. Implementing the Analysis with Volatility Futures

In order to evaluate a potential impact of early rollover for VSTOXX futures, we therefore compared the performance of all VSTOXX futures time series (i.e. 1-month, 3-month and LT time series) when rolled over 5 days prior to maturity versus that when rolled over at maturity. Unlike VIX futures, the final settlement price of VSTOXX futures contracts is estimated as an average of actual VSTOXX values on the last trading day between 11:30 and 12:00 CET, therefore, reducing the possibility of arbitrage trading. Although rolling over 5 days prior to maturity slightly reduced the volatility of futures series, we found no evidence that it significantly improved the results in portfolios with long volatility exposure. Considering this result, we use rollover at maturity in all our further empirical analyses.

Figure 13 depicts the relative performance of the VSTOXX Index and all VSTOXX futures time series (i.e. 1-month, 3-month and LT time series) over the sample period. As was the case for the VSTOXX Index in the previous section, all futures series are negatively correlated with the EURO STOXX 50 index. The correlation between EURO STOXX 50 and the 1-month, 3-month and LT futures time series' returns are -0.69, -0.68 and -0.61, respectively. It should be noted that the 1-month series exhibits the highest negative correlation with EURO STOXX index; however it has the largest drag on performance due to higher rollover costs. In contrast, the longest-traded (LT) series lead to lower trading costs, but also show a lower negative correlation with the EURO STOXX 50 index. The 3-month series

![Figure 13: Performance of EURO STOXX 50 Index and VSTOXX Futures](image)
3. Implementing the Analysis with Volatility Futures

The performance of a long volatility position implemented with VSTOXX futures differs from a hypothetical situation where the ‘theoretical’ investment in volatility is assumed to be made by directly investing in VSTOXX Index (as discussed in the previous section). The VSTOXX return is driven by changes in the level of implied volatilities. In contrast, the returns of the VSTOXX futures are driven by changes in expectations of implied volatilities (Dash and Moran 2007; Szado 2009; Alexander and Korovilas 2011). The relationship is further complicated by the fact that volatility tends to follow a mean reverting process. Due to the mean reverting nature of volatility, the investment in VSTOXX futures is a priori expected to exhibit a significantly lower volatility than the ‘theoretical’ direct investment in VSTOXX Index. This is confirmed in our sample where, for example, the 3-month VSTOXX Futures series has a very similar return to the VSTOXX index (-7.78% p.a. vs. -7.76% p.a.) over the sample period with substantially lower volatility (61.1% p.a. as compared to 101.3% of VSTOXX Index).

3.2. Portfolio Analysis

In order to be consistent with our earlier analysis of adding a long volatility exposure to the equity portfolio through the VSTOXX index, we again construct several equity portfolios with a few different allocations to a VSTOXX futures position.

As before, the European equity market exposure will be approximated by the investment in EURO STOXX 50 Index. The investment in VSTOXX futures is represented by a long position in a fully collateralised VSTOXX futures position. To create this investment, a long position in the front-month futures contract is fully collateralised by holding a full value of the contract in a bank deposit paying EURIBOR interest rate. By the end of the day, all positions are rebalanced by marking-to-market and adjusting the collateral position to reflect the cash inflow or outflow from marking to market. Returns for all days between the roll-in day and maturity date are calculated using the mid-point between the bid and ask prices. The futures position is rolled into the next contract at the close on the day prior to maturity.

We analyse the carry and rollover costs of buy-and-hold VIX futures positions based on different rollover methodologies (1-month, 3-month and LT time series). First, we consider an ‘efficient’ investment in VSTOXX futures with no associated bid-ask spread costs – for instance, by assuming that each contract is rolled over to the next one based on the mid-price defined as the mid-point between the bid and ask prices. As in the previous section, we constructed a number of equity portfolios with increasing allocations to VSTOXX futures. The analysis starts with the pure equity portfolio as a benchmark case and adds, in 5% increments, a long volatility exposure to the portfolio.

Given that both EURO STOXX 50 and VSTOXX futures returns were negative over the sample period, we have adjusted...
3. Implementing the Analysis with Volatility Futures

the performance measure used to compare the portfolios. Due to negative returns, a traditional Sharpe ratio would result in negative values that are difficult to interpret. We used a similar measure, however, instead of calculating excess returns over a risk-free rate, we estimated them over a specific benchmark, which in this case was a pure equity portfolio EURO STOXX 50 Index return. This measure indicates how well each portfolio performed when comparing to the benchmark case, adjusted for the riskiness of the portfolio.

The summary table with the results of this analysis is presented in Appendix 1. The 3-month VSTOXX Futures series were a clear winner in this case. All diversified portfolios with allocations to 3-month VSTOXX futures series outperformed a pure equity portfolio over the sample period. The 3-month series performed significantly better than 1-month series where allocation to volatility exposure managed to reduce portfolio volatility, but failed to improve the returns. In addition, 3-month series showed better results than longest-traded (LT) series as well - the maximum ‘adjusted’ performance measure achieved with 3-month series totalled to 0.41, as compared to 0.26 with LT series.

The results for the best performing 3-month VSTOXX futures series are presented in Figure 14 in an efficient frontier format. Clearly, adding an exposure with 3-month VSTOXX futures series is beneficial to the portfolio performance in both return and volatility terms. It is interesting to note that the best performing portfolio is achieved by allocating 30% to VSTOXX futures, which is a similar result to that obtained with VSTOXX index data (see Figure 5). For more detailed comparison, the results for all VSTOXX Futures series (1-month, 3-month, LT) in efficient frontier form are included in Appendix 2.

Figure 14: Impact of Adding Long Volatility Exposure to Equity Portfolio in 5% Increments

The effects of adding VSTOXX Futures (the 3-month series) to the EURO STOXX 50 portfolio in 5% increments, estimated based on the sample period ranging from April 2008 to April 2011.
3. Implementing the Analysis with Volatility Futures

3.3. Transaction Costs
When considering futures as an instrument for a long volatility exposure, we however also need to take into consideration actual trading costs (i.e. bid-ask spread) that are associated with the futures’ rollover strategy. In order to access the full impact of transaction costs, we incorporated the bid-ask spread into the analysis. In this case, all long VSTOXX futures positions are rolled into at their ask prices then are closed at bid prices on the day prior to maturity. The results of the analysis are presented in Appendix 3. Including the bid-ask spread, costs significantly affect the performance of VSTOXX futures. The returns are decreased by 26.6%, 12.6% and 8.5% p.a. for 1-month, 3-month and longest-term (LT) futures, respectively. After accounting for bid-ask spread costs, adding the 1-month series no longer contributes to improving the performance over the pure equity case. The 3-month and longest-traded (LT) series still show some potential for improving portfolio diversification; however, the effect is mostly due to reduced volatility of the overall portfolio with little if any improvement in average performance.

In order to simulate the impact of trading costs over a longer sample period, we applied average costs associated with trading VSTOXX futures to VSTOXX Index data that is available for the sample period ranging from January 1999 to April 2011. We used the maximum Sharpe ratio portfolio that was estimated with VSTOXX Index data in the previous section and adjusted its performance for the costs that would be incurred if VSTOXX futures

![Figure 15: Performance of EURO STOXX 50 Index and Diversified Portfolio with VSTOXX Futures Series](image-url)

Daily time series for EURO STOXX 50 Index and diversified portfolios with VSTOXX Futures series on the sample period ranging from January 1999 to April 2011.
3. Implementing the Analysis with Volatility Futures

were to actually be traded. The results are presented in Figure 15, and confirm the benefits of introducing a long volatility exposure even after the presence of trading costs are accounted for.

3.4. Comparison with a Global Minimum Variance Portfolio

We also compare the performance of the diversified portfolio with a managed volatility position with VSTOXX futures to one of the GMV portfolio. As in the previous section, MSCI Europe Minimum Volatility index is used as a proxy for the GMV portfolio and the diversified portfolio has 70% allocation to EURO STOXX 50 and 30% allocation to VSTOXX exposure. We used the best-performing 3-month VSTOXX futures series to create an investment in VSTOXX index.

As indicated in Figure 16, the diversified portfolio with VSTOXX Futures presents a better investment opportunity than GMV portfolio. Firstly, it would reduce the standard deviation of returns from 21.6% (for GMV portfolio) to 16.1% (for diversified portfolio). Secondly, it would also reduce negative returns for the sample period – from -2.8% (GMV) to -1.0% (diversified portfolio).

We can also take a closer look at both GMV and diversified portfolios with VSTOXX futures exposure performance during 2008 financial crisis (see Figure 17). Both portfolios exhibit rather similar performance until the middle of September 2008, when unprecedented losses in the market occurred, following the failure of Lehman Brothers. After that, the performance of GMV portfolio deteriorated sharply resulting to 28.2% loss in portfolio value. The diversified portfolio with VSTOXX futures exposure managed to protect the investment value and even gain 7.4% due to significantly increased volatility in the market. The volatility of the diversified portfolio returns was also lower at 19.3% as compared to 31.0% volatility of GMV portfolio.

Figure 16: Performance of Diversified Portfolio with VSTOXX Futures and Global Minimum Variance (GMV) Portfolio

Daily time series for the diversified portfolio and MSCI Europe Minimum Volatility Index on the sample period ranging from April 2008 to April 2011.
In more recent periods, starting from January 2010, the comparison of performances of GMV portfolio and diversified portfolio with VSTOXX futures exposure is more complicated (see Figure 18). The diversified portfolio with VSTOXX futures exposure clearly outperformed the GMV portfolio in bear market that started in May 2001, however in the following less volatile period it decreased in value more than the GMV portfolio (−6.7% p.a. vs. 6.8% for the GMV portfolio). This is mainly due to negative carry and roll yield of VSTOXX futures, considering that
3. Implementing the Analysis with Volatility Futures

A portfolio with a ‘theoretical’ investment in VSTOXX Index (see Figure 9) would achieve much better results during this period.

In summary, the results obtained in this section suggest that the benefits of adding a long volatility exposure to equity portfolios, which are particularly strong during market downturns, are robust with respect to the introduction of trading costs involved in implementation with volatility futures contracts. Careful attention to trade execution is nonetheless required to limit the negative impact of transaction costs, negative carry and roll yield on volatility futures during normal periods.
4. Short-term Analysis with Volatility Options
4. Short-term Analysis with Volatility Options

In this section, we consider a different approach based on the use of volatility options for gaining a long exposure to volatility. March 2010 witnessed the introduction of option contracts on the VSTOXX index, which provided investors with more flexibility for trading European volatility.

The data on VSTOXX put/call options is collected from the intraday transaction records provided by Eurex and Bloomberg.

4.1. Portfolio Analysis with VSTOXX Calls

While futures contracts are the most straightforward approach to gaining long volatility exposure, using options might allow investors to gain access to the upside of volatility exposure without the associated downside. Empirical results in several recent papers (Grant et al. 2007; Szado 2009) actually suggest that call options have potential to provide a particularly effective diversification of equity risk compared to other financial instruments (e.g. volatility futures or index puts).

It should be noted that the data history available for VSTOXX options is very short (ranging from March 2010 to April 2011). Due to an extremely short data history and corresponding sample size, it would be difficult to provide a formal analysis of the marginal benefits to be expected from using volatility index futures as opposed to volatility index options. Therefore, the analysis in this section is merely to be regarded as an example of an alternative way for structuring a long volatility exposure.

In order to assess the impact of adding VSTOXX options to equity portfolios, we constructed the portfolio strategy by rolling over one month to expiration VSTOXX call options. We use both at-the-money (ATM) and out-of-the-money (10% OTM and 25% OTM) calls for our analysis. Each month, on the day before expiration, the options are rolled to the next month contract and the portfolio is rebalanced to the target weights. Considering that volatility options are much more sensitive to changes in underlying volatility compared to fully collateralised futures contracts, we used 1% increments in volatility exposure rather than 5% increments used for VSTOXX futures.

The results of the analysis are presented in Appendix 4. The performance of ATM VSTOXX calls provides very similar results to the VSTOXX futures. While adding a small positive exposure to the volatility index option portfolio (1% and 2%) slightly improves the performance of the overall portfolio, further increases provide no additional value. Due to increased leverage, the results achieved with OTM calls is much more favourable than those achieved with ATM calls; and in the case of the 25% OTM calls, the return improvements are impressive.

The performance of the portfolios with increasing allocation to 25% OTM calls is depicted in an efficient frontier form in Figure 19. The portfolio with the best performance (as compared with an all equity case) is with 2% allocation to OTM VSTOXX calls and the rest (98%) allocated to equity. It should be noted that the diversified portfolio with a similar volatility of returns to that of the equity portfolio
4. Short-term Analysis with Volatility Options

Figure 19: Impact of Adding VSTOXX 25% OTM options to Equity Portfolio in 1% Increments

The effects of adding VSTOXX OTM options to EURO STOXX 50 portfolio by 1% increments, estimated based on the sample period ranging from March 2010 to April 2011.

(19.4% p.a. vs. 22.8% p.a.) performed significantly better in terms of returns – while all equity portfolios resulted in 0.95% p.a. return, the diversified portfolio with OTM call exposure had an extremely attractive return of 12.3% p.a. during this period. For comparison, the results of the portfolios with allocation to all VSTOXX option series (i.e. ATM, 10% OTM and 25% OTM) are presented in the Appendix 5.

4.2. Transaction Costs

In further analysis, we estimated the impact of the bid-ask spread on the performance of the diversified portfolios with VSTOXX options exposure. In this case VSTOXX options are rolled in at the ask price at the open and rolled out at intrinsic value. The results for all series are presented in Appendix 6. When bid-ask spread costs are taken into consideration, adding ATM calls to the portfolio does not increase the returns, although, they still reduce standard deviation of the portfolio.

However, the benefits of adding OTM (both 10% and 25%) calls are still clear, even after including transaction costs in the analysis. In both cases, holding a 2% allocation to volatility options ex-post led to the highest "adjusted" Sharpe ratio. Compared to pure equity portfolio return of 0.95% during this period, the portfolios with 2% allocations to 10% OTM and 25% OTM calls resulted in 5.2% p.a. and 15.8% p.a. returns respectively. Also, the volatility is even decreased with 10% OTM call allocation equalling to 17.12% as compared with 22.8% of equity portfolio.

4.3. Protective EURO STOXX 50 puts

In this section, we compare the results obtained so far with the results that would be generated by a classic strategy for managing downside risk in equity portfolios – the use of protective puts. In every financial textbook, protective puts are referred to as a direct hedge for the price movements in equity portfolios.
4. Short-term Analysis with Volatility Options

Although, holding a long EURO STOXX 50 put position is a simple way to eliminate equity portfolio losses in the declining market, there is empirical evidence (e.g. Szado 2009) that the opportunity might be prohibitive because of the smile effect (implied volatility of out-of-the-money puts tends to be higher than the implied volatility of in-the-money puts).

In previous chapters, we proved that the negative correlation between EURO STOXX 50 and VSTOXX is conditional in nature and tends to be strongest in the large downward market moves, when it is needed the most. Considering these conditions, one might expect that the diversification benefits of VSTOXX options may provide a more efficient way to manage downside risk than EURO STOXX 50 puts.

In order to test this conjecture, we compare the performance of an equity portfolio with VSTOXX call allocations to that of an equity portfolio mixed with long EURO STOXX 50 puts. For consistent analysis, we use the same moneyness levels for EURO STOXX 50 puts as we used for VSTOXX calls (i.e. ATM, 10% OTM and 25% OTM). Each month, at the same rollover day as used for VSTOXX calls, EURO STOXX 50 put positions are rolled over to the next month contract. ATM, 10% OTM and 25% OTM puts are added to the base equity portfolio using 1% increments.

The performance of portfolios mixed with VSTOXX calls was analysed in section 4.1 and the results presented in Appendix 4. The summary statistics of the performance of portfolios with EURO STOXX 50 puts for the sample period ranging from March 2010 to April 2011 is presented in Appendix 7. We find that equity portfolios with EURO STOXX 50 put positions do not perform as well as portfolios mixed with VSTOXX calls. None of the portfolios with EURO STOXX 50 puts have better ‘adjusted’ Sharpe ratios than those of a pure equity portfolio. While the allocation to ATM puts helps to reduce volatility (although at the expense of decreasing returns), allocation to OTM puts mostly increases volatility and results in losses for the portfolio.

Overall, there seems to be an efficiency gain on our sample period from using VSTOXX derivatives or futures for downside risk protection on equity portfolios compared to using EUROSTOXX 50 put options.

4.4. Using Volatility Derivatives to Bet on Volatility Changes

Up to this point, we have mostly focused on the diversification properties of volatility derivatives. However, an investor can also use VSTOXX options to trade on a specific view on the VSTOXX direction or volatility changes. In this section, we analyse the performance of two commonly used strategies for generating premium: (i) short out-of-the-money VSTOXX puts; and (ii) VSTOXX ratio spread strategy.

4.4.1. Shorting out-of-the-money VSTOXX puts

In order to implement this strategy, an investor can sell VSTOXX puts on a systematic basis if a rise in volatility levels is expected. If expected volatility is increasing, this position allows premium income to be generated, while also creating a long volatility exposure.
4. Short-term Analysis with Volatility Options

However, its performance can be poor if volatility starts declining after a large spike.

We first test this strategy by estimating the position’s performance of selling the next downside puts on each monthly rollover day for the sample period ranging from March 2010 to April 2011. This position is then added to the equity portfolio in 1% increments to evaluate its effectiveness in overall portfolio management. The results (presented in Appendix 8) indicate that although this strategy slightly reduced overall portfolio volatility (from 22.8% p.a. of equity portfolio to 19.4% p.a. with 2% investment in short VSTOXX puts), it did not improve the returns. This is due to decreasing VSTOXX levels during this period, as selling downside put strikes commonly resulted in a loss. All premium income earned was eroded as volatility dropped following May sovereign crisis in 2010.

An alternative way for implementing this strategy includes a more careful selection of put strikes. In this case, out-of-the-money VSTOXX puts are sold on a ‘tiered’ basis. We used the following rules for selecting out-of-the-money VSTOXX puts.

This approach allows for picking deeper out-of-the-money puts when volatility level is high, and it is therefore more likely to mean-revert back to a lower level. By applying this approach, it turns out that all short VSTOXX put positions expire out of the money and a full premium is earned. As a result, it significantly improved the performance of the strategy based on shorting out-of-the-money VSTOXX put options. A summary statistic for an equity portfolio with a 1% increasing allocation to short VSTOXX puts is presented in Appendix 8. A short VSTOXX put positioned with carefully selected strikes during the sample period resulted in a significant profit with each additional 1% of VSTOXX put allocation, resulting in over 10% p.a. return increase (although it was somehow less effective in reducing volatility).

The improvement in overall portfolio performance driven by using the ‘tiered’ approach for selecting short VSTOXX put strikes is substantial. For example, a pure equity portfolio earns 0.95% p.a. return with 22.8% p.a. volatility for the sample period, and the portfolio with 2% short VSTOXX put allocation, that has a similar 22.8% volatility, results in an impressive 24.4% p.a. return over the same period.

4.4.2. A ratio call spread strategy
The ratio spread is a strategy that involves buying a number of options and selling more options of the same underlying stock at expiration and at a different strike price. This strategy is consistent with the view that the underlying asset (in this case, a volatility index) will

<table>
<thead>
<tr>
<th>VSTOXX Front Month Futures</th>
<th>VSTOXX Put Strike with Distance to Future</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;25</td>
<td>5</td>
</tr>
<tr>
<td>25-30</td>
<td>7.5</td>
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<tr>
<td>30-35</td>
<td>10</td>
</tr>
<tr>
<td>35-40</td>
<td>15</td>
</tr>
</tbody>
</table>
experience little volatility in the near term. Hence, the ratio call spread strategy is used to get upside exposure to volatility at a reduced premium outlay. While this position benefits from 'modest' increases in VSTOXX levels, it is, however exposed to the risk of a large volatility spike.

Implied volatility of VSTOXX options is typically quite high (at levels of 80%) and, in addition, the call skew can be quite steep. We can capitalise upon this by selling two deep out-of-the-money VSTOXX calls and using the proceeds to purchase a closer out-of-the-money call. As in the previous section, we tested this strategy with two approaches. First, on each monthly rollover day, two 25% OTM calls are sold and one closer OTM call with the strike difference of 5 volatility points is bought. The second approach tested is a ‘tiered’ approach, where we still sell two 25% OTM calls, but the strike of a purchased closer OTM call is selected based on the following rules.

By applying a ‘tiered’ approach, we would sell further out-of-the-money calls in low volatility regime to avoid losing on the first two short calls if VSTOXX increases sharply, while, in high volatility regime, the overall increase in volatility of volatility would impact the call skew and provide more premium for two sold deep out-of-the-money calls.

In each case, the call ratio spread position was added to the equity portfolio in 1% increments. The results for both strategies are presented in Appendix 9. We can conclude that the first strategy with a fixed 5 volatility points difference between strikes was, somehow, useful in reducing volatility (from 22.8% p.a. of equity portfolio to 16.0% p.a. with 2% investment in short VSTOXX puts), however, it resulted in significant losses for the portfolio over the sample period.

The second ‘tiered’ approach was more successful in improving return, however, at the expense of increased overall volatility of the portfolio. For example, the equity portfolio with 2% allocation to the call ratio spread generates a return of 4.2% p.a. with 36.4% p.a. volatility over the sample period. In comparison, an equity portfolio with 2% allocation to pure 25% OTM VSTOXX call position earns 22.0% p.a. return with 30.8% p.a. volatility.

We introduced both short out-of-the-money VSTOXX puts and ratio call strategies as examples of how VSTOXX options can be used in more innovative ways for the equity portfolio management. In both cases, a careful selection of option strike prices proved to be critical for portfolio performance. Therefore, it is important to take current market volatility conditions into account when designing and implementing an option trading strategy.

4. Short-term Analysis with Volatility Options

<table>
<thead>
<tr>
<th>VSTOXX Front Month Futures</th>
<th>VSTOXX Call Strike with Distance to Future</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;25</td>
<td>10</td>
</tr>
<tr>
<td>25-35</td>
<td>7.5</td>
</tr>
<tr>
<td>&gt;35</td>
<td>5</td>
</tr>
</tbody>
</table>
In this paper, we analyse a novel approach in the design of attractive equity solutions with managed volatility, based on mixing a well-diversified equity portfolio with volatility derivatives, as opposed to minimising equity volatility through minimum variance approaches. The results we obtain suggest that a long volatility position shows a strongly negative correlation with respect to the underlying equity portfolio and that adding a long volatility exposure to an equity portfolio would result in a substantial improvement of the risk-adjusted performance of the portfolio. The benefits of the long volatility exposure are found to be strongest in market downturns, when they are most needed.

We also compare the performance of the diversified equity portfolios including volatility derivatives with that of global minimum variance (GMV) portfolios that are commonly used in the industry as a benchmark strategy for reducing portfolio risk. We found that the diversified portfolio with long volatility exposure is a more efficient approach for managing risk.

We also consider the challenges related to a practical implementation of this strategy by using derivatives instruments – futures and options – that allow investors direct access to trading volatility. We consider how increasing allocation to volatility derivatives affects the portfolio performance; we also evaluate transaction costs in each case and discuss the advantages/disadvantages for using each type of instrument. The benefits of adding volatility exposure to equity portfolios are found to be robust with respect to the introduction of trading costs associated with rolling over volatility derivatives contracts.

5. Conclusion
Appendix
Appendix

Appendix 1
Summary Statistics for Portfolios with Increasing Allocation to VSTOXX Futures

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td></td>
<td>Annualised Return</td>
<td>Annualised Std Deviation</td>
<td>Adj. Sharpe Ratio*</td>
</tr>
<tr>
<td>100% EURO STOXX 50</td>
<td>-7.47%</td>
<td>29.48%</td>
<td>-0.03</td>
</tr>
<tr>
<td>95% EURO STOXX 50 + 5% VSTOXX Futures</td>
<td>-6.68%</td>
<td>24.80%</td>
<td>0.03</td>
</tr>
<tr>
<td>90% EURO STOXX 50 + 10% VSTOXX Futures</td>
<td>-6.25%</td>
<td>21.18%</td>
<td>0.06</td>
</tr>
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<td>85% EURO STOXX 50 + 15% VSTOXX Futures</td>
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<td>18.87%</td>
<td>0.07</td>
</tr>
<tr>
<td>80% EURO STOXX 50 + 20% VSTOXX Futures</td>
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<td>18.13%</td>
<td>0.06</td>
</tr>
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<td>75% EURO STOXX 50 + 25% VSTOXX Futures</td>
<td>-6.84%</td>
<td>18.95%</td>
<td>0.03</td>
</tr>
<tr>
<td>70% EURO STOXX 50 + 30% VSTOXX Futures</td>
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<td>21.04%</td>
<td>-0.01</td>
</tr>
<tr>
<td>65% EURO STOXX 50 + 35% VSTOXX Futures</td>
<td>-8.63%</td>
<td>23.99%</td>
<td>-0.048</td>
</tr>
<tr>
<td>60% EURO STOXX 50 + 40% VSTOXX Futures</td>
<td>-9.88%</td>
<td>27.47%</td>
<td>-0.09</td>
</tr>
<tr>
<td>55% EURO STOXX 50 + 45% VSTOXX Futures</td>
<td>-13.14%</td>
<td>73.84%</td>
<td>-0.36</td>
</tr>
<tr>
<td>50% EURO STOXX 50 + 50% VSTOXX Futures</td>
<td>-34.14%</td>
<td>78.11%</td>
<td>-0.38</td>
</tr>
</tbody>
</table>

*Note: Due to negative returns, a traditional Sharpe ratio would result in negative values that are difficult to interpret. We adjusted the performance measure, where, instead of calculating excess returns over a risk-free rate, we estimated them over a specific benchmark (i.e. EURO STOXX 50 Index return). This measure indicates how well each portfolio performed when compared to all equity portfolio cases, adjusted for the riskiness of the portfolio.
Appendix 2
Impact of Adding VSTOXX Index Futures exposure to Equity portfolio in 5% increments
Appendix 3
Summary Statistics for Portfolios with Increasing Allocation to VSTOXX Futures (Including Bid-Ask Spread Costs)

| Jun/2009 – Apr/2011 | 100% EURO STOXX 50 | 95% EURO STOXX 50 + 5% VSTOXX Futures | 90% EURO STOXX 50 + 10% VSTOXX Futures | 85% EURO STOXX 50 + 15% VSTOXX Futures | 80% EURO STOXX 50 + 20% VSTOXX Futures | 75% EURO STOXX 50 + 25% VSTOXX Futures | 70% EURO STOXX 50 + 30% VSTOXX Futures | 65% EURO STOXX 50 + 35% VSTOXX Futures | 60% EURO STOXX 50 + 40% VSTOXX Futures | 55% EURO STOXX 50 + 45% VSTOXX Futures | 50% EURO STOXX 50 + 50% VSTOXX Futures | 45% EURO STOXX 50 + 55% VSTOXX Futures | 40% EURO STOXX 50 + 60% VSTOXX Futures | 35% EURO STOXX 50 + 65% VSTOXX Futures | 30% EURO STOXX 50 + 70% VSTOXX Futures | 25% EURO STOXX 50 + 75% VSTOXX Futures | 20% EURO STOXX 50 + 80% VSTOXX Futures | 15% EURO STOXX 50 + 85% VSTOXX Futures | 10% EURO STOXX 50 + 90% VSTOXX Futures | 5% EURO STOXX 50 + 95% VSTOXX Futures | 0% VSTOXX Futures |
|---------------------|--------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| 1-Month Futures Time Series: | | | | | | | | | | | | | | | | | | | | | | | | | |
| Ann. Return | -7.47% | -9.01% | -10.89% | -13.06% | -15.48% | -18.12% | -20.93% | -23.89% | -26.96% | -29.09% | -31.22% | -33.35% | -35.48% | -37.61% | -39.74% | -41.87% | -43.99% | -46.12% | -48.25% | -50.38% | -52.51% | -54.64% |
| Ann. Std Deviation | 29.48% | 24.88% | 21.34% | 19.09% | 18.39% | 19.23% | 21.32% | 24.29% | 27.79% | 31.28% | 34.77% | 38.26% | 41.75% | 45.24% | 48.73% | 52.22% | 55.71% | 59.20% | 62.69% | 66.18% | 69.67% |
| Adj. Sharpe Ratio* | - | -0.06 | -0.16 | -0.29 | -0.44 | -0.55 | -0.63 | -0.68 | -0.70 | -0.72 | -0.74 | -0.74 | -0.74 | -0.74 | -0.74 | -0.74 | -0.74 | -0.74 | -0.74 | -0.74 | -0.74 |
| 3-Month Futures Time Series: | | | | | | | | | | | | | | | | | | | | | | | | | |
| Ann. Return | -7.47% | -6.50% | -5.77% | -5.26% | -4.94% | -4.82% | -4.88% | -5.11% | -5.51% | -5.91% | -6.31% | -6.71% | -7.11% | -7.51% | -7.91% | -8.31% | -8.71% | -9.11% | -9.51% | -9.91% | -10.31% |
| Ann. Std Deviation | 29.48% | 25.02% | 21.27% | 18.31% | 16.35% | 15.56% | 15.99% | 17.48% | 19.74% | 22.00% | 24.26% | 26.52% | 28.78% | 31.04% | 33.30% | 35.56% | 37.82% | 39.08% | 41.34% | 43.60% |
| Adj. Sharpe Ratio* | - | 0.04 | 0.08 | 0.12 | 0.15 | 0.17 | 0.16 | 0.14 | 0.10 | -0.19 | -0.21 | -0.23 | -0.25 | -0.27 | -0.29 | -0.31 | -0.33 | -0.35 | -0.37 | -0.39 |
| LT Futures Time Series: | | | | | | | | | | | | | | | | | | | | | | | | | |
| Ann. Return | -7.47% | -6.71% | -6.19% | -5.89% | -5.80% | -5.89% | -6.15% | -6.54% | -7.17% | -7.90% | -8.54% | -9.17% | -9.80% | -10.44% | -11.07% | -11.70% | -12.33% | -12.96% | -13.59% | -14.22% |
| Ann. Std Deviation | 29.48% | 25.35% | 21.95% | 19.32% | 17.57% | 16.77% | 16.95% | 17.99% | 19.68% | 21.36% | 23.04% | 24.72% | 26.40% | 28.08% | 29.76% | 31.44% | 33.12% | 34.80% | 36.48% |
| Adj. Sharpe Ratio* | - | 0.03 | 0.06 | 0.08 | 0.09 | 0.09 | 0.08 | 0.06 | 0.02 | -0.24 | -0.29 | -0.34 | -0.39 | -0.44 | -0.49 | -0.54 | -0.59 | -0.64 | -0.69 | -0.74 |

*Note: Due to negative returns, a traditional Sharpe ratio would result in negative values that are difficult to interpret. We adjusted the performance measure, where, instead of calculating excess returns over a risk-free rate, we estimated them over a specific benchmark (i.e. EURO STOXX 50 Index return). This measure indicates how well each portfolio performed when compared to all equity portfolio cases, adjusted for the riskiness of the portfolio.
### Appendix 4

**Summary Statistics for Portfolios with Increasing Allocation to VSTOXX Calls**

<table>
<thead>
<tr>
<th>Mar/20010 – Apr/2011</th>
<th>100% EURO STOXX 50</th>
<th>99% EURO STOXX 50 + 1% VSTOXX Options</th>
<th>98% EURO STOXX 50 + 2% VSTOXX Options</th>
<th>97% EURO STOXX 50 + 3% VSTOXX Options</th>
<th>96% EURO STOXX 50 + 4% VSTOXX Options</th>
<th>95% EURO STOXX 50 + 5% VSTOXX Options</th>
<th>94% EURO STOXX 50 + 6% VSTOXX Options</th>
<th>93% EURO STOXX 50 + 7% VSTOXX Options</th>
<th>92% EURO STOXX 50 + 8% VSTOXX Options</th>
<th>91% EURO STOXX 50 + 9% VSTOXX Options</th>
<th>100% VSTOXX Options</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATM Calls :</td>
<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Ann. Return</td>
<td>0.95%</td>
<td>1.29%</td>
<td>0.96%</td>
<td>0.02%</td>
<td>-1.46%</td>
<td>-3.43%</td>
<td>-5.80%</td>
<td>-8.53%</td>
<td>-11.56%</td>
<td>-14.83%</td>
<td>-17.43%</td>
</tr>
<tr>
<td>Ann. Std Deviation</td>
<td>22.81%</td>
<td>18.03%</td>
<td>16.01%</td>
<td>17.12%</td>
<td>20.48%</td>
<td>24.95%</td>
<td>29.86%</td>
<td>34.92%</td>
<td>39.98%</td>
<td>44.99%</td>
<td>777.31%</td>
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<tr>
<td>Adj. Sharpe Ratio*</td>
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<td>0.02</td>
<td>0.00</td>
<td>-0.05</td>
<td>-0.12</td>
<td>-0.23</td>
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<td>10% OTM Calls :</td>
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<tr>
<td>Ann. Return</td>
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<td>5.80%</td>
<td>9.53%</td>
<td>12.19%</td>
<td>13.84%</td>
<td>14.57%</td>
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<td>11.96%</td>
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<td>-100%</td>
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<td>Ann. Std Deviation</td>
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<td>18.29%</td>
<td>23.82%</td>
<td>30.93%</td>
<td>38.37%</td>
<td>45.74%</td>
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<td>66.46%</td>
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<td>Adj. Sharpe Ratio*</td>
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<td>0.47</td>
<td>0.47</td>
<td>0.42</td>
<td>0.35</td>
<td>0.30</td>
<td>0.24</td>
<td>0.18</td>
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<tr>
<td>Ann. Return</td>
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<td>22.03%</td>
<td>30.12%</td>
<td>36.60%</td>
<td>41.54%</td>
<td>45.02%</td>
<td>47.13%</td>
<td>47.97%</td>
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<td>Ann. Std Deviation</td>
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*Note: Due to negative returns, a traditional Sharpe ratio would result in negative values that are difficult to interpret. We adjusted the performance measure, where, instead of calculating excess returns over a risk-free rate, we estimated them over a specific benchmark (i.e. EURO STOXX 50 Index return). This measure indicates how well each portfolio performed when compared to all equity portfolio cases, adjusted for the riskiness of the portfolio.*
Appendix

Appendix 5
Impact of Adding VSTOXX Index Options exposure to Equity portfolio in 1% increments
## Appendix 6

### Summary Statistics for Portfolios with Increasing Allocation to VSTOXX Calls (With Bid-Ask Spread Costs)

|          | Mar/20010 – Apr/2011 | 100% EURO STOXX 50 | 99% EURO STOXX 50 + 1% VSTOXX Options | 98% EURO STOXX 50 + 2% VSTOXX Options | 97% EURO STOXX 50 + 3% VSTOXX Options | 96% EURO STOXX 50 + 4% VSTOXX Options | 95% EURO STOXX 50 + 5% VSTOXX Options | 94% EURO STOXX 50 + 6% VSTOXX Options | 93% EURO STOXX 50 + 7% VSTOXX Options | 92% EURO STOXX 50 + 8% VSTOXX Options | 91% EURO STOXX 50 + 9% VSTOXX Options | 100% VSTOXX Options |
|----------|----------------------|---------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------|
| **ATM Calls** |                      |                     |                                        |                                        |                                        |                                        |                                        |                                        |                                        |                                        |                                        |                                        |                      |
| Ann. Return | 0.95%                | 0.76%               | -0.04%                                 | -1.38%                                 | -3.21%                                 | -5.46%                                 | -8.07%                                 | -10.99%                                | -14.18%                                | -17.59%                                | -21.00%                                | -25.50%                                | -100.00%             |
| Ann. Std Deviation | 22.81%               | 18.27%              | 16.00%                                 | 16.44%                                 | 19.07%                                 | 22.92%                                 | 27.32%                                 | 31.95%                                 | 36.64%                                 | 41.30%                                 | 785.92%                                | 785.92%                                |                      |
| Adj. Sharpe Ratio* | -                    | -0.01               | -0.06                                  | -0.14                                  | -0.22                                  | -0.28                                  | -0.33                                  | -0.37                                  | -0.41                                  | -0.45                                  | -0.13                                  | -0.13                                  |                      |
| **10% OTM Calls** |                      |                     |                                        |                                        |                                        |                                        |                                        |                                        |                                        |                                        |                                        |                                        |                      |
| Ann. Return | 0.95%                | 3.51%               | 5.15%                                  | 5.95%                                  | 5.97%                                  | 5.29%                                  | 3.97%                                  | 2.1%                                   | -0.27%                                 | -3.05%                                 | -100.00%                                | -100.00%                                |                      |
| Ann. Std Deviation | 22.81%               | 17.67%              | 17.10%                                 | 20.60%                                 | 26.11%                                 | 32.30%                                 | 38.64%                                 | 44.94%                                 | 51.11%                                 | 57.12%                                 | 776.10%                                | 776.10%                                |                      |
| Adj. Sharpe Ratio* | -                    | 0.14                | 0.25                                   | 0.24                                   | 0.19                                   | 0.13                                   | 0.08                                   | 0.03                                   | -0.02                                  | -0.07                                  | -0.13                                  | -0.13                                  |                      |
| **25% OTM Calls** |                      |                     |                                        |                                        |                                        |                                        |                                        |                                        |                                        |                                        |                                        |                                        |                      |
| Ann. Return | 0.95%                | 9.52%               | 15.82%                                 | 20.16%                                 | 22.80%                                 | 23.97%                                 | 23.87%                                 | 22.70%                                 | 20.62%                                 | 17.78%                                 | -100.00%                                | -100.00%                                |                      |
| Ann. Std Deviation | 22.81%               | 20.39%              | 28.20%                                 | 39.39%                                 | 51.09%                                 | 62.59%                                 | 73.73%                                 | 84.47%                                 | 94.84%                                 | 104.8%                                 | 1034.10%                               | 1034.10%                               |                      |
| Adj. Sharpe Ratio* | -                    | 0.42                | 0.53                                   | 0.49                                   | 0.43                                   | 0.37                                   | 0.31                                   | 0.26                                   | 0.21                                   | 0.16                                   | -0.10                                  | -0.10                                  |                      |

*Note: Due to negative returns, a traditional Sharpe ratio would result in negative values that are difficult to interpret. We adjusted the performance measure, where, instead of calculating excess returns over a risk-free rate, we estimated them over a specific benchmark (i.e. EURO STOXX 50 Index return). This measure indicates how well each portfolio performed when compared to all equity portfolio cases, adjusted for the riskiness of the portfolio.
## Appendix 7

### Summary Statistics for Portfolios with Increasing Allocation to EURO STOXX 50 Puts

| Mar/20010 – Apr/2011 | 100% EURO STOXX 50 | 99% EURO STOXX 50 + 1% VSTOXX Options | 98% EURO STOXX 50 + 2% VSTOXX Options | 97% EURO STOXX 50 + 3% VSTOXX Options | 96% EURO STOXX 50 + 4% VSTOXX Options | 95% EURO STOXX 50 + 5% VSTOXX Options | 94% EURO STOXX 50 + 6% VSTOXX Options | 93% EURO STOXX 50 + 7% VSTOXX Options | 92% EURO STOXX 50 + 8% VSTOXX Options | 91% EURO STOXX 50 + 9% VSTOXX Options | 100% VSTOXX Options |
|----------------------|-------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|
| **ATM Puts:**        |                   |                                      |                                      |                                      |                                      |                                      |                                      |                                      |                                      |                                      |                                      |
| Ann. Return          | 0.99%             | 0.33%                                | -0.68%                               | -2.04%                               | -3.71%                               | -5.15%                               | -7.94%                               | -10.24%                              | -12.81%                              | -15.54%                              | -18.54%                              | -21.77%                              |
| Ann. Std Deviation   | 22.81%            | 17.77%                               | 13.65%                               | 11.15%                               | 11.19%                               | 13.08%                               | 16.91%                               | 20.00%                               | 25.31%                               | 29.79%                               | 34.02%                               | 537.62%                              |
| Adj. Sharpe Ratio*   | -0.03             | -0.12                                | -0.27                                | -0.42                                | -0.49                                | -0.52                                | -0.53                                | -0.54                                | -0.55                                | -0.55                                | -0.55                                | -0.19                                |
| **10% OTM Puts:**    |                   |                                      |                                      |                                      |                                      |                                      |                                      |                                      |                                      |                                      |                                      |                                      |
| Ann. Return          | 0.99%             | -1.50%                               | -4.34%                               | -7.52%                               | -10.95%                              | -14.57%                              | -18.34%                              | -22.20%                              | -26.11%                              | -30.03%                              | -34.00%                              | -100%                                |
| Ann. Std Deviation   | 22.81%            | 16.31%                               | 19.50%                               | 27.56%                               | 36.63%                               | 45.68%                               | 54.49%                               | 63.02%                               | 71.28%                               | 79.90%                               | 89.16%                               | 912.51%                              |
| Adj. Sharpe Ratio*   | -0.15             | -0.27                                | -0.31                                | -0.32                                | -0.34                                | -0.35                                | -0.37                                | -0.38                                | -0.39                                | -0.38                                | -0.38                                | -0.11                                |
| **25% OTM Puts:**    |                   |                                      |                                      |                                      |                                      |                                      |                                      |                                      |                                      |                                      |                                      |                                      |
| Ann. Return          | 0.99%             | -7.88%                               | -16.03%                              | -23.51%                              | -30.39%                              | -36.73%                              | -42.54%                              | -47.87%                              | -52.75%                              | -57.22%                              | -61.50%                              | -100%                                |
| Ann. Std Deviation   | 22.81%            | 19.71%                               | 27.84%                               | 38.93%                               | 50.26%                               | 61.28%                               | 71.89%                               | 82.09%                               | 91.90%                               | 101.43%                              | 116.25%                              | 116.25%                              |
| Adj. Sharpe Ratio*   | -0.46             | -0.61                                | -0.63                                | -0.62                                | -0.61                                | -0.60                                | -0.59                                | -0.58                                | -0.57                                | -0.57                                | -0.57                                | -0.10                                |

*Note: Due to negative returns, a traditional Sharpe ratio would result in negative values that are difficult to interpret. We adjusted the performance measure, where, instead of calculating excess returns over a risk-free rate, we estimated them over a specific benchmark (i.e. EURO STOXX 50 Index return). This measure indicates how well each portfolio performed when compared to all equity portfolio cases, adjusted for the riskiness of the portfolio.
## Appendix 8

**Summary Statistics for Portfolios with Increasing Allocation to Short VSTOXX Puts**

### Appendix

The Benefits of Volatility Derivatives in Equity Portfolio Management — May 2012

| Mar/2010–Apr/2011 | 100% EURO STOXX 50 | 99% EURO STOXX 50 + 1% VSTOXX Options | 98% EURO STOXX 50 + 2% VSTOXX Options | 97% EURO STOXX 50 + 3% VSTOXX Options | 96% EURO STOXX 50 + 4% VSTOXX Options | 95% EURO STOXX 50 + 5% VSTOXX Options | 94% EURO STOXX 50 + 6% VSTOXX Options | 93% EURO STOXX 50 + 7% VSTOXX Options | 92% EURO STOXX 50 + 8% VSTOXX Options | 91% EURO STOXX 50 + 9% VSTOXX Options | 100% VSTOXX Options |
|-------------------|-------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------|
| **Ann. Return**   | 0.95%             | -1.92%                              | -4.85%                              | -7.85%                              | -10.90%                             | -13.99%                             | -17.12%                             | -20.27%                             | -23.45%                             | -26.63%                             | -29.84%                             | -33.03%           |
| **Ann. Std Deviation** | 22.81%            | 21.10%                              | 19.93%                              | 19.41%                              | 19.62%                              | 20.53%                              | 22.08%                              | 24.15%                              | 26.64%                              | 29.45%                              | n/a                  | n/a               |
| **Adj. Sharpe Ratio** | -                 | -0.14                               | -0.29                               | -0.45                               | -0.60                               | -0.73                               | -0.82                               | -0.88                               | -0.92                               | -0.94                               | n/a                  | n/a               |

### Next Strike Down:

| **Ann. Return** | 0.95% | 12.11% | 24.39% | 37.88% | 52.69% | 63.94% | 86.75% | 106.3% | 127.6% | 150.96% | n/a                | n/a               |
| **Ann. Std Deviation** | 22.81% | 21.56% | 21.81% | 23.63% | 26.72% | 30.85% | 35.72% | 41.15% | 47.06% | 53.37% | n/a                | n/a               |
| **Adj. Sharpe Ratio** | -     | 0.52   | 1.07   | 1.56   | 1.94   | 2.20   | 2.40   | 2.56   | 2.69   | 2.81   | n/a                | n/a               |

### Tiered Approach:

*Note: Due to negative returns, a traditional Sharpe ratio would result in negative values that are difficult to interpret. We adjusted the performance measure, where, instead of calculating excess returns over a risk-free rate, we estimated them over a specific benchmark (i.e. EURO STOXX 50 Index return). This measure indicates how well each portfolio performed when compared to all equity portfolio cases, adjusted for the riskiness of the portfolio.*
### Appendix 9

**Summary Statistics for Portfolios with Increasing Allocation to Ratio Spread Strategy**

| Mar/20010 – Apr/2011 | 100% EURO STOXX 50 | 99% EURO STOXX 50 + 1% VSTOXX Options | 98% EURO STOXX 50 + 2% VSTOXX Options | 97% EURO STOXX 50 + 3% VSTOXX Options | 96% EURO STOXX 50 + 4% VSTOXX Options | 95% EURO STOXX 50 + 5% VSTOXX Options | 94% EURO STOXX 50 + 6% VSTOXX Options | 93% EURO STOXX 50 + 7% VSTOXX Options | 92% EURO STOXX 50 + 8% VSTOXX Options | 91% EURO STOXX 50 + 9% VSTOXX Options | 100% VSTOXX Options |
|----------------------|-------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|
| **Strike Difference - 5 vol points:** | | | | | | | | | | | | |
| **Ann. Return** | 0.95% | -39.24% | -83.81% | n/a | n/a | n/a | n/a | n/a | n/a | n/a | n/a | n/a |
| **Ann. Std Deviation** | 22.81% | 18.03% | 16.01% | n/a | n/a | n/a | n/a | n/a | n/a | n/a | n/a | n/a |
| **Adj. Sharpe Ratio*** | - | -0.36 | -0.16 | n/a | n/a | n/a | n/a | n/a | n/a | n/a | n/a | n/a |
| **Tiered** Approach : | | | | | | | | | | | | |
| **Ann. Return** | 0.95% | 2.65% | 4.18% | 5.54% | 6.71% | 7.67% | 8.42% | 8.93% | 9.20% | 9.21% | n/a |
| **Ann. Std Deviation** | 22.81% | 28.32% | 36.36% | 45.59% | 55.45% | 65.69% | 76.25% | 87.11% | 98.29% | 109.83% | n/a |
| **Adj. Sharpe Ratio*** | - | 0.06 | 0.09 | 0.10 | 0.10 | 0.10 | 0.09 | 0.08 | 0.08 | n/a |

*Note: Due to negative returns, a traditional Sharpe ratio would result in negative values that are difficult to interpret. We adjusted the performance measure, where, instead of calculating excess returns over a risk-free rate, we estimated them over a specific benchmark (i.e. EURO STOXX 50 Index return). This measure indicates how well each portfolio performed when compared to all equity portfolio cases, adjusted for the riskiness of the portfolio.*
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About EDHEC-Risk Institute
About EDHEC-Risk Institute

The Choice of Asset Allocation and Risk Management
EDHEC-Risk structures all of its research work around asset allocation and risk management. This issue corresponds to a genuine expectation from the market. On the one hand, the prevailing stock market situation in recent years has shown the limitations of diversification alone as a risk management technique and the usefulness of approaches based on dynamic portfolio allocation. On the other, the appearance of new asset classes (hedge funds, private equity, real assets), with risk profiles that are very different from those of the traditional investment universe, constitutes a new opportunity and challenge for the implementation of allocation in an asset management or asset-liability management context.

This strategic choice is applied to all of the Institute’s research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; taking extreme risks into account in portfolio construction; studying the usefulness of derivatives in implementing asset-liability management approaches; or orienting the concept of dynamic “core-satellite” investment management in the framework of absolute return or target-date funds.

Academic Excellence and Industry Relevance
In an attempt to ensure that the research it carries out is truly applicable, EDHEC has implemented a dual validation system for the work of EDHEC-Risk. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the Institute’s advisory board. This board is made up of internationally recognised researchers, the Institute’s business partners, and representatives of major international institutional investors. Management of the research programmes respects a rigorous validation process, which guarantees the scientific quality and the operational usefulness of the programmes.

Six research programmes have been conducted by the centre to date:
- Asset allocation and alternative diversification
- Style and performance analysis
- Indices and benchmarking
- Operational risks and performance
- Asset allocation and derivative instruments
- ALM and asset management

These programmes receive the support of a large number of financial companies. The results of the research programmes are disseminated through the EDHEC-Risk locations in Singapore, which was established at the invitation of the Monetary Authority of Singapore (MAS), the City of London in the United Kingdom, and Nice, France. In addition, it has a research team located in the United States.

EDHEC-Risk has developed a close partnership with a small number of sponsors within the framework of research chairs or major research projects:
- Core-Satellite and ETF Investment, in partnership with Amundi ETF
- Regulation and Institutional Investment, in partnership with AXA Investment Managers
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- Asset-Liability Management and Institutional Investment Management, *in partnership with BNP Paribas Investment Partners*
- Risk and Regulation in the European Fund Management Industry, *in partnership with CACEIS*
- Exploring the Commodity Futures Risk Premium: Implications for Asset Allocation and Regulation, *in partnership with CME Group*
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- Structured Equity Investment Strategies for Long-Term Asian Investors, *in partnership with Société Générale Corporate & Investment Banking*

The philosophy of the Institute is to validate its work by publication in international academic journals, as well as to make it available to the sector through its position papers, published studies, and conferences.

Each year, EDHEC-Risk organises two conferences for professionals in order to present the results of its research, one in London (EDHEC-Risk Days – Europe) and one in Singapore (EDHEC-Risk Days – Asia), attracting more than 2,000 professional delegates.

EDHEC also provides professionals with access to its website, www.edhec-risk.com, which is entirely devoted to international asset management research. The website, which has more than 50,000 regular visitors, is aimed at professionals who wish to benefit from EDHEC’s analysis and expertise in the area of applied portfolio management research. Its monthly newsletter is distributed to more than 1,000,000 readers.

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The EDHEC-Risk Institute PhD in Finance
The EDHEC-Risk Institute PhD in Finance is designed for professionals who aspire to higher intellectual levels and aim to redefine the investment banking and asset management industries. It is offered in two tracks: a residential track for high-potential graduate students, who hold part-time positions at EDHEC, and an executive track for practitioners who keep their full-time jobs. Drawing its faculty from the world’s best universities and enjoying the support of the research centre with the greatest impact on the financial industry, the EDHEC-Risk Institute PhD in Finance creates an extraordinary platform for professional development and industry innovation.

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