

Volatility – The Perfect Asset Class for the Equity Fund Manager?

“Volatility should be considered as an equity sector in its own right alongside financials, industrials and mining stocks”, Emmanuel Bourdeix, Head of Derivatives & Convertibles, Credit Agricole Asset Management, Paris – “Volatility: Good Time to Sail with the Wind”, Financial Times.

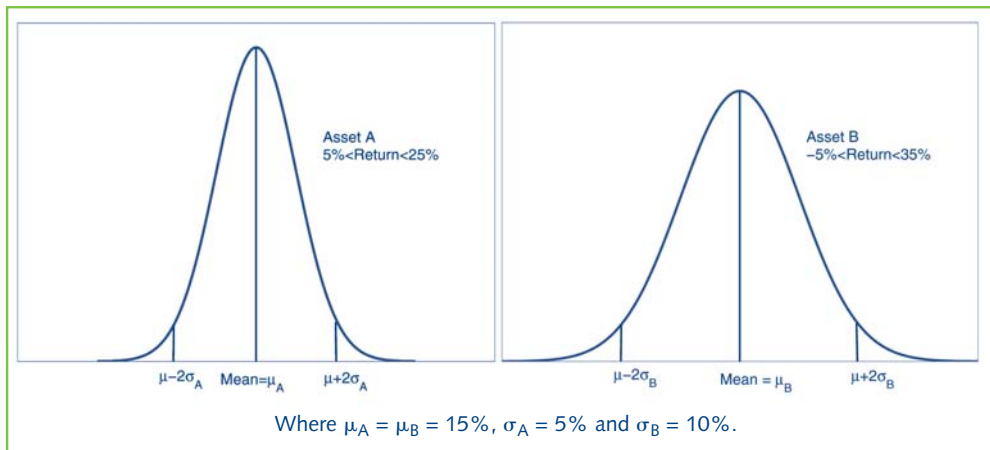
Introduction

On September 19, 2005 Eurex launched three exchange-traded volatility index futures contracts¹, on the VSTOXX®, the VDAX-NEW® and the VSMI®, which for their part are based on the leading benchmark European equity index options contracts of the Dow Jones EURO STOXX 50®, DAX® and SMI® respectively. In January 2006, the Committee for European Securities Regulators clarified that volatility derivative products were UCITS-compliant, extending the “tool box” of products available to the equity fund manager.

What is Volatility?

Volatility, expressed notionally as σ , is a measure of the uncertainty of the returns provided by an asset. Statistically, it is the *standard deviation* of the returns provided by the asset (*variance*, another measure of dispersion, is the square of volatility i.e. σ^2). Volatility gives a measure of the probability distribution of the future returns of an asset. For example, two assets, A and B, both give the same *average* return of 15%, yet asset A has a standard deviation (i.e. volatility) of 5% and asset B has a standard deviation of 10% – asset B is a more *volatile* asset than A despite both assets giving the same average return. Assuming a normal distribution, statistical theory says that over time there is a 95% probability of an assets’ return falling within two standard deviations (i.e. 2σ) of the mean, μ (i.e. the average). The higher the standard deviation (i.e. the higher the volatility) the greater the probability/expectation of higher and lower returns of the asset. Diagram 1 below looks at the probability distribution of the return of the two assets, A and B, based on their differing volatilities/standard deviations:

Diagram 1: Expected Returns and Volatility



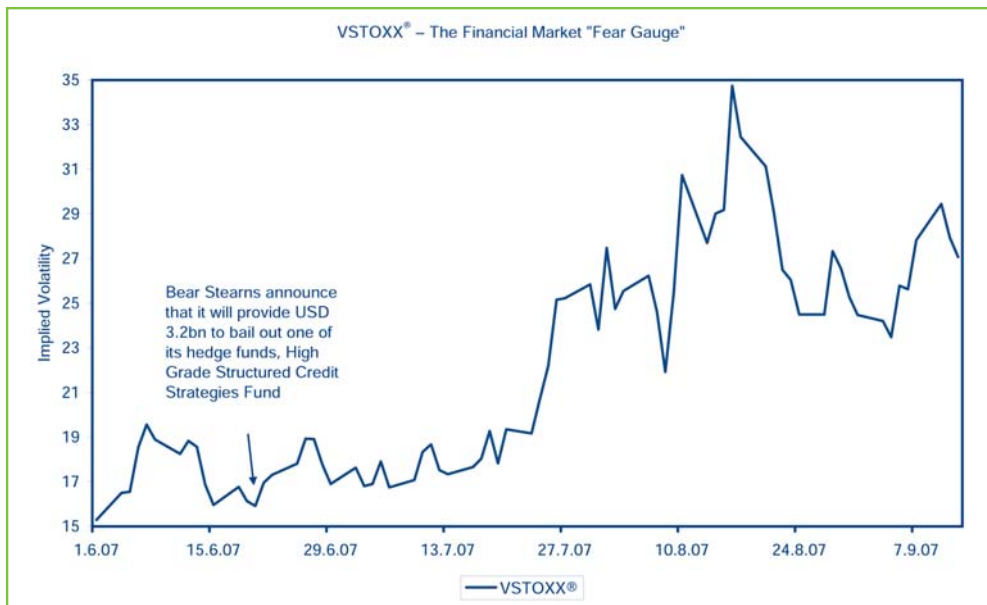
¹ See Appendix 1, Eurex Volatility Index futures – contract specifications.

With a probability of 95% asset B would be expected to have a much higher range of possible returns (i.e. -5% to 35%) with a standard deviation of 10% to that of asset A with a standard deviation of 5% and with expected returns of between 5% and 25%.

There are three types of volatility, *historical*, *implied* and *realized*. Historical volatility measures what volatility *has been* over a given time period. Implied volatility is the volatility *implied* in the option price given the other variables that is time, strike price, underlying asset price, etc. Realized volatility is the *subsequent movement* in the assets' volatility following the transaction.

It is not possible to calculate implied volatility using the Black-Scholes model² but it is calculated *iteratively* using the *Newton-Raphson technique*. Implied volatility has the characteristic that options for different strikes but identical expiration dates trade at different implied volatilities – this phenomenon is known as the *implied volatility smile* (or skew). Implied volatility will also have a *term structure* in that for the same strike price but different expiration dates options will have different implied volatilities. Volatility (and the shape of the volatility smile/skew) has always been regarded as reflecting market participants' expectations and being a "lead indicator" of impending moves in the underlying asset^{3, 4} – the Eurex volatility Indexes can be used as a barometer of the state of the financial market i.e. a "fear gauge"⁵. Diagram 2 below depicts how VSTOXX[®] has responded to the recent crisis in financial markets – the index rising *before* the full crisis erupted:

Diagram 2: Eurex Volatility Index Futures – The European Financial Markets' "Fear Gauge"



² However, Corrado and Miller in "Volatility without Tears", Risk magazine, have produced a formula for implied volatility under the Black-Scholes model which is accurate across a range of underlying asset prices.

³ A number of articles have been written on the predictive power of options and the volatility skew, namely, B. Mizraeh, "Did option prices predict the ERM Crisis?", R. Cont, "Beyond implied volatility: Extracting information from option prices" and G. Murphy, "When option prices meet the volatility smile".

⁴ S. Chadwick, Credit Suisse, in "Can the VIX Signal Market Direction?" looked at the predictive power of the VIX[®], the U.S. volatility index contract. Studies by Giot, Guo and *Active Trader* magazine also found relationships between VIX[®] and future stock market returns.

⁵ See Robert E. Whalley, "The Investor Fear Gauge".

The Relationship Between Equity Volatility and Equity Markets

The basis of the attraction of Eurex volatility indexes to the equity fund manager lies in the **negative correlation of equity volatility to the equity market** offering increased portfolio diversification and reduced portfolio risk. The causality between equity market and equity volatility can be attributed to "the leverage effect" – a fall in equity prices increases a company's leverage, thereby increasing the risk to equity holders, and increasing equity volatility. John C. Hull in "Options, Futures & Other Derivatives", outlined the causality as follows – "As a company's equity declines in value, the company's leverage increases. This means that the equity becomes more risky and its volatility increases. As a company's equity increases in value, leverage decreases. The equity then becomes less risky and its volatility decreases. This argument shows that we can expect the volatility of equity to be a decreasing function of price." Diagrams 3 below look at the historical statistical relationship between (daily) changes in the spot volatility indexes that is VSTOXX®, VDAX-NEW® and VSMI® and (daily) changes in their respective equity indexes that is Dow Jones EURO STOXX 50®, DAX® and SMI® between July 2006 and February 2007:

Diagram 3a: Relationship Between DAX® and VDAX-NEW®

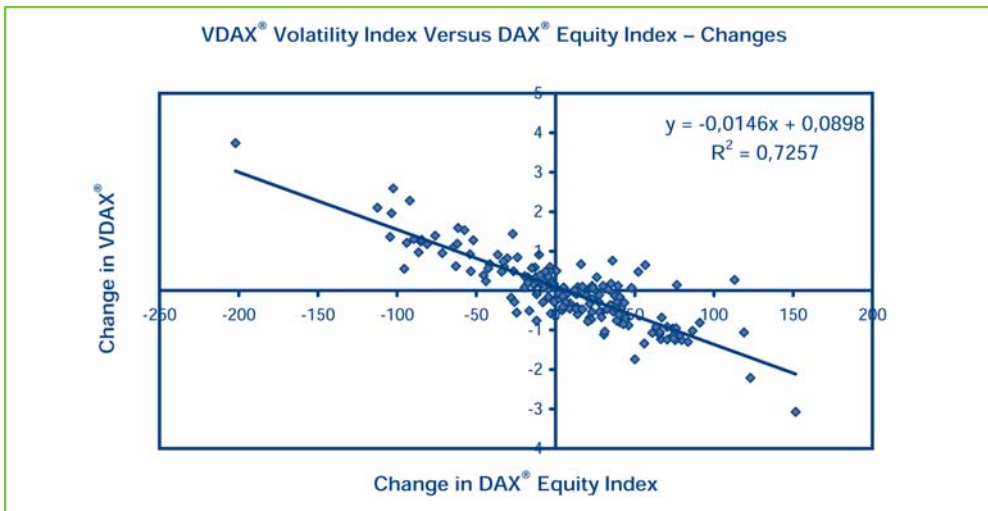


Diagram 3b: Relationship Between Dow Jones EURO STOXX 50® and VSTOXX®

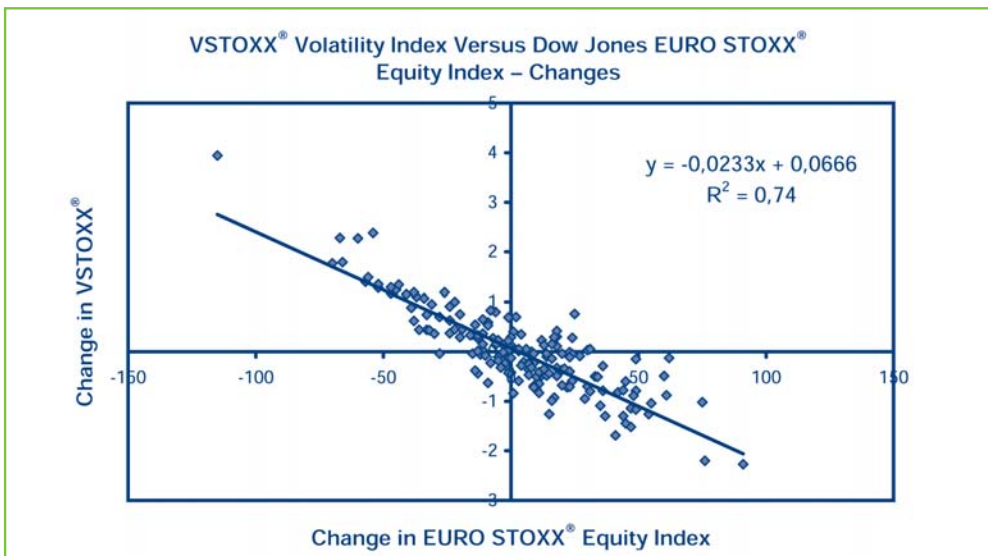
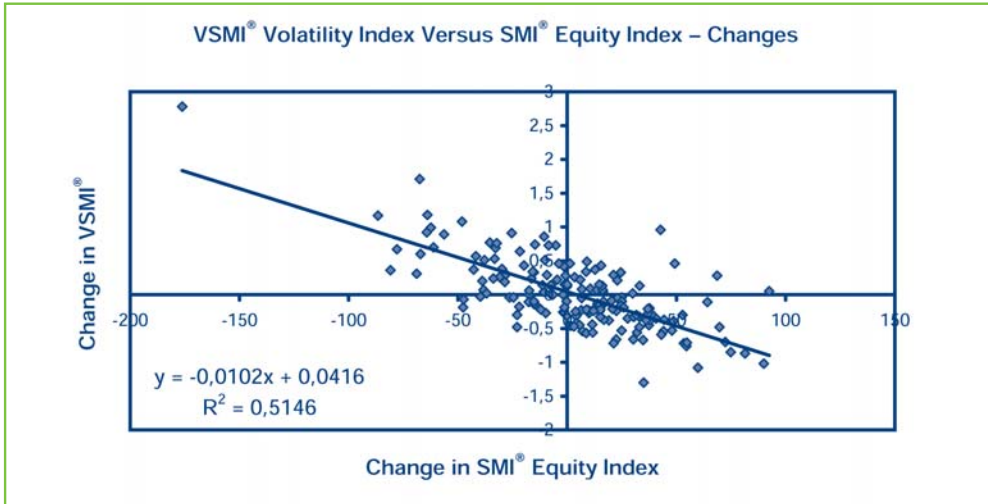


Diagram 3c: Relationship Between SMI® and VSMI®



What do the results in Diagrams 3a, 3b and 3c show? With regards to Diagram 3a, based on the period under analysis, a 100 index points decrease in the DAX® Equity Index price is accompanied with a $(-0.0146)(100) = 1.46\%$ rise in implied volatility in the VDAX® Volatility Index. Similarly, there was found to be a similar inverse relationship for VSTOXX®/Dow Jones EURO STOXX 50® Index and VSMI®/SMI® Index a decrease of 100 index points fall in Dow Jones EURO STOXX 50® Index is accompanied with a corresponding $(-0.0233)(100) = 2.33\%$ increase in VSTOXX®, similarly, a 100 point index fall in SMI® Index Futures is accompanied by a corresponding $(-0.0102)(100) = 1.02\%$ rise in VSMI®.

Eurex Volatility Futures

The Eurex volatility futures contracts⁶ on the VSTOXX®, the VDAX-NEW® and the VSMI® represent an implied volatility level on the underlying equity index options of Dow Jones EURO STOXX 50®, DAX® and SMI® respectively. The implied volatility levels of the Eurex volatility futures contracts take into consideration the volatility skew and corresponds to the square root of implied variance that is $\sqrt{\sigma^2}$ (in fact Eurex volatility index futures are 100. $\sqrt{\sigma^2}$ for the quote to reflect volatility in percentage terms) to bring the quotation close to that used to evaluate variance in the OTC variance swap market. The formula for the computation of the volatility index is not based on a particular options pricing model like Black-Scholes and makes volatility more tradeable as an asset with changes in the value of the contract due to changes in volatility and not due to any changes in the underlying equity market – if one VSTOXX® Future was bought at 14.00 (%) and sold at 16.00 (%) then the payoff would be the index multiplier times the difference between the two volatilities that is $\text{EUR } 1,000 \times (16-14) = \text{EUR } 2,000$ – a straightforward linear payoff to a change in volatility.

Volatility and Variance – Payoff Profiles

The payoff to volatility and variance are different.

In a variance swap the payoff is expressed as:

$$\text{Payoff (Variance Swap)} = \text{Notional} \times (\text{Realized Implied Volatility}^2 - \text{Implied Volatility}^2)$$

⁶ See Appendix 2 – Calculation of Deutsche Boerse AG Volatility Indexes.

For example, a EUR 5 million variance swap⁷ struck at an implied volatility (squared) of 50% with an eventual realized volatility of 70% would have a payoff, to the buyer of the swap, of EUR 5 million \times (70%² – 50%²) = EUR 1,200,000. Alternatively, if at the end of the swap period implied volatility had fallen to 30% the loss would have been EUR 5 million \times (50%² – 30%²) = EUR 800,000. The payoff profile in a variance swap is not symmetrical but convex – the payoff is similar to a long call option profile.

With regards to the Eurex volatility index futures contracts the payoff of a future held can be written as:

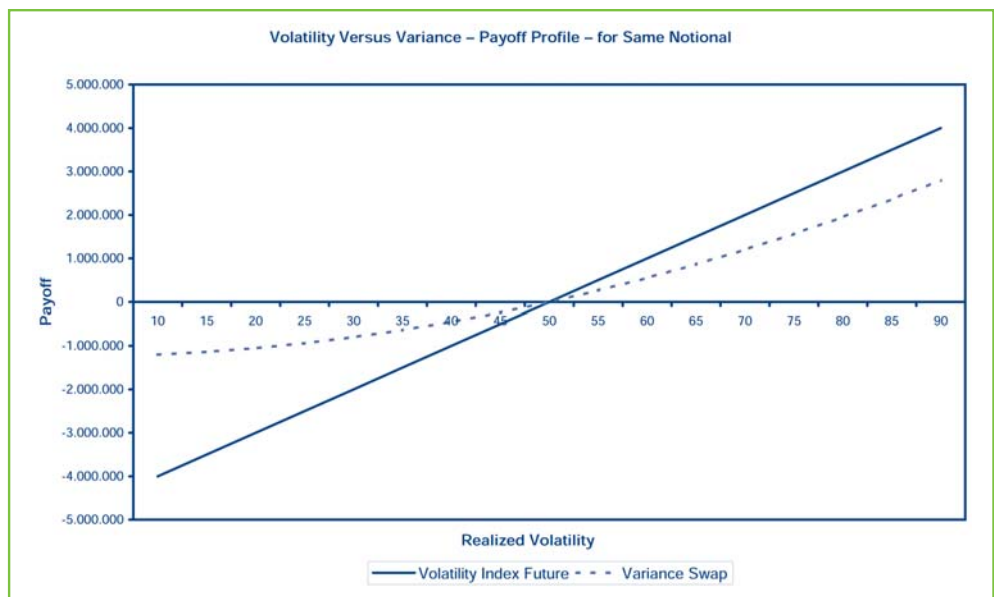
$$\text{Payoff (Volatility Index Futures)} = \text{Index Multiplier (i.e. EUR 1,000 or CHF 1,000)} \times (\text{Realized 30-day Implied Volatility Level at Expiration} - \text{Expected 30-day Implied Volatility at trade initiation}) \times \text{Number of Contracts}$$

In contrast to the variance swap example, a EUR 5 million position (i.e. 100 contracts) in VSTOXX[®] volatility index Futures struck at an expected implied volatility of 50% with an eventual implied volatility at maturity of 70%, the payoff to the buyer would be EUR 1,000 \times (70% – 50%) \times 100 = EUR 2,000,000. Alternatively, if implied volatility fell to 30% the loss would be EUR 1,000 \times (50% – 30%) \times 100 = EUR 2,000,000. The payoff is linear – **a Eurex volatility index future can be likened to that of a forward on 30-day (implied) volatility⁷** and it can be compared to that of a volatility swap whose payoff profile can be expressed as follows:

$$\text{Payoff (Volatility Swap)} = \text{Notional} \times (\text{Realized Volatility} - \text{Implied Volatility})$$

Where the level of implied volatility is set at the initiation of the trade and the realized volatility is attained at the maturity of the swap. Diagram 4 below compares the two different payoff profiles of being long variance through a variance swap and long volatility through a long position in future on VSTOXX[®]:

Diagram 4: Volatility Versus Variance – Payoff Profile – for Same Notional



⁷ Variance and volatility swaps are actually forward contracts.

Fund Management Applications of Eurex Volatility Index Futures

Increased Portfolio Diversification and Enhanced Portfolio Returns

The negative correlation of equity volatility to its respective equity market makes equity volatility an attractive asset to incorporate within an equity portfolio to increase portfolio diversification and potentially enhance portfolio returns. An analysis was carried out to determine the effects of incorporating equity volatility into an equity portfolio by taking the respective equity indexes of Dow Jones EURO STOXX[®], DAX[®] and SMI[®] as the “core” equity index portfolio and combining them with their respective volatility indexes of VSTOXX[®], VDAX-NEW[®] and VSMI[®] in a 80%/20% weighting (weighted in terms of monetary value) adjusted weekly over a period from June 28, 1999 to August 15, 2007. The results are shown in Diagrams 5a, 5b and 5c below:

Diagram 5a: Dow Jones EURO STOXX 50[®] Index Portfolio Versus Combined 80%/20% Dow Jones EURO STOXX 50[®]/VSTOXX[®] Index Portfolio

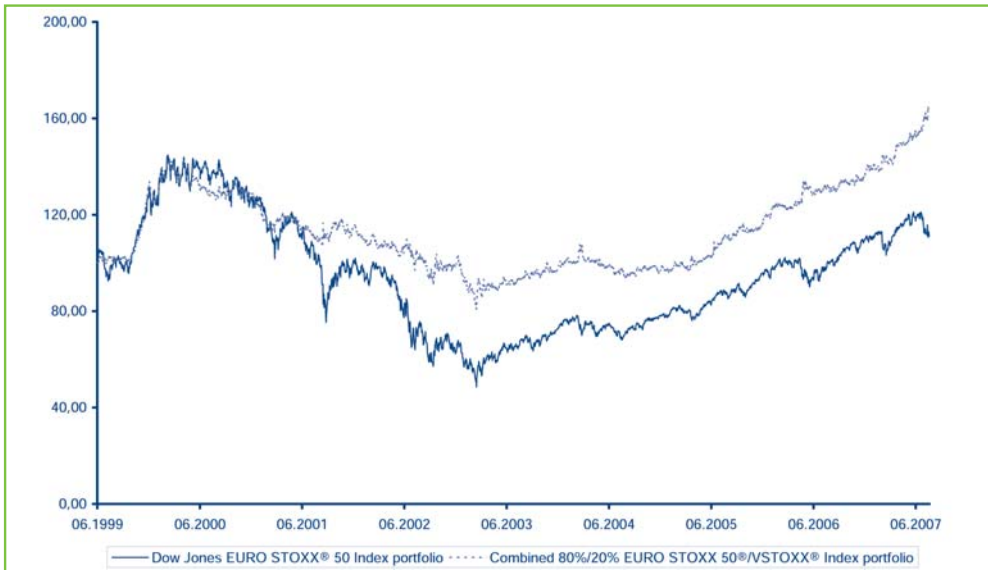
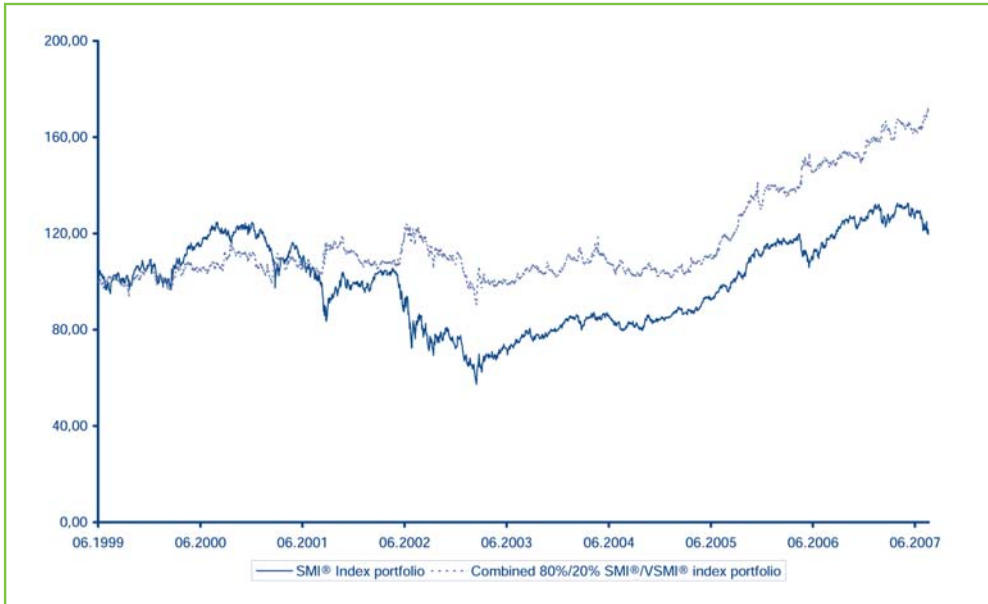


Diagram 5b: DAX[®] Index Portfolio Versus Combined 80%/20% DAX[®]/VDAX-NEW[®] Index Portfolio



Diagram 5c: SMI® Index Portfolio versus Combined 80%/20% SMI®/VSMI® Index Portfolio



As the above diagrams show, under the period of analysis, the combined 80%/20% equity/volatility index outperformed the underlying equity index as represented by its respective equity index in **all** the equity markets under analysis. **A study to maximize the return of the combined volatility/stock portfolio for the period under analysis, i.e. June 28, 1999 to August 15, 2007, for each of the markets under observation, generated optimum allocations of 53.3% DAX®/ 46.7% VDAX®, 46.2% Dow Jones EURO STOXX®/ 53.8% VSTOXX® and 55.2% SMI®/ 44.8% VSMI® underlying the benefits of incorporating volatility into an equity portfolio.**

Managing Tracking Error and Rebalancing Costs

Benchmark/passive index equity fund managers are in essence, *short volatility*. As equity markets become more volatile, tracking error and rebalancing costs increase. Equity fund managers can go long Eurex volatility index futures to hedge against increases in portfolio tracking error and rebalancing costs of its benchmark/passive index funds. Similarly, convertible bond arbitrage fund managers can use Eurex volatility index futures to hedge their imbedded volatility exposure. Periods of low dispersion/high correlation across equities make it difficult for fund managers to extract alpha in stock selection, buying Eurex volatility index futures could be used to hedge low dispersion/high correlation.

Generating Alpha

Volatility Versus Credit

Because of the leverage effect, already outlined above, one would expect an *inverse relationship* between equity prices and credit spreads – or a *positive relationship* between the Eurex iTraxx® CDS index future and equity (there could be situations i.e. LBO and M&A activity, which would result in a breakdown of the inverse relationship and actually result in increasing equity prices with *increasing* credit spreads) and a *positive relationship* between credit spreads and equity volatility or a *negative relationship* between iTraxx® CDS futures price and equity volatility. Diagrams 6a and 6b outline the relationship between September iTraxx® CDS Crossover futures and the VDAX® and VSTOXX® volatility indices respectively:

Diagram 6a: iTraxx® CDS Crossover Futures and VDAX®

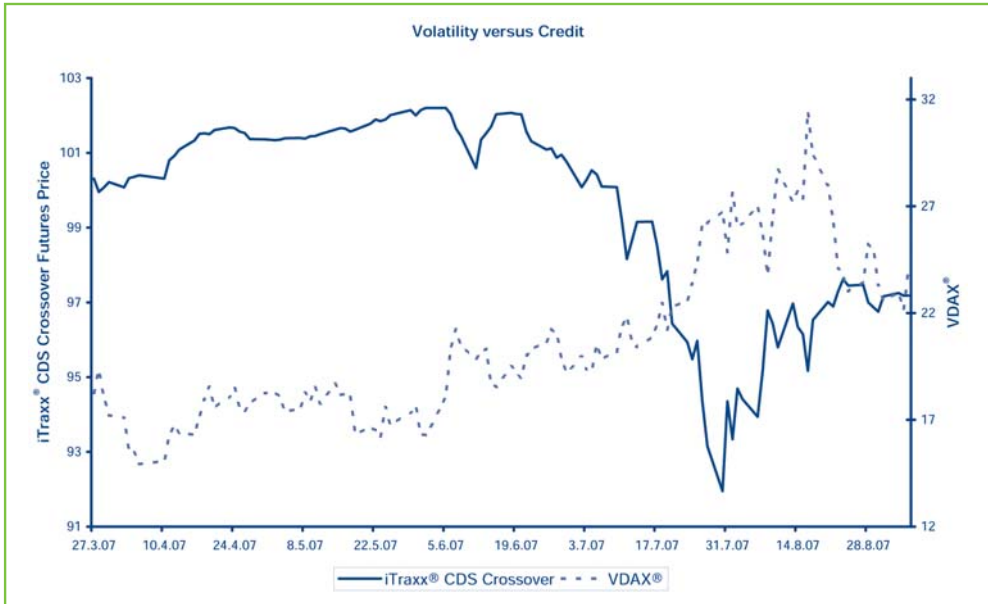
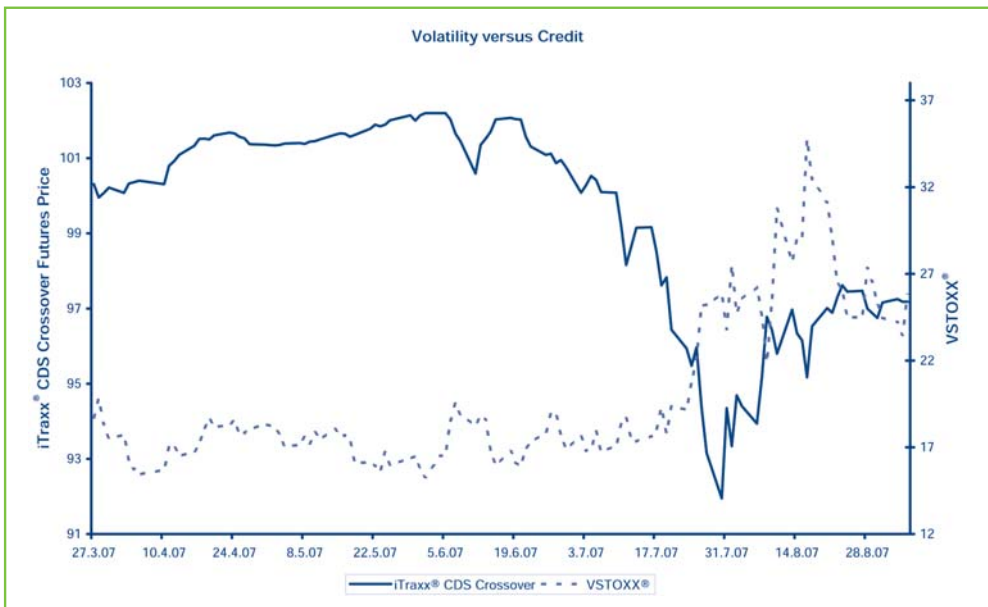


Diagram 6b: iTraxx® CDS Crossover Futures and VSTOXX®



How would you structure an equity volatility versus credit trade through a Eurex iTraxx® CDS Crossover/ VDAX® spread position? One method would be to structure such strategies in terms of the ratio of the monetary value of each of the respective contracts' risk positions based on historical price volatility:

iTraxx® CDS Crossover Future

$96.68 \text{ (price)} \times 13.825\% \text{ (30-day historical price volatility)} = 13.36 = \text{EUR } 13,360.$

VDAX®

$26.05 \text{ (price)} \times 92.25\% \text{ (30-day historical price volatility)} = 24.03 = \text{EUR } 24,030.$

Therefore, based on the contracts' respective risk positions based on 30-day historical price volatility, **the ratio of iTraxx® CDS Crossover Futures to VDAX® Volatility Futures** would be **1.80:1** (obviously, as historical price volatility changes, the ratio would need to be adjusted).

Cheap Vega Positioning and Volatility Spreads

Eurex volatility index futures offer a very *cheap* and *leveraged* way to initiate volatility directional strategies as there is no requirement to delta hedge for movement in the underlying asset – Axel Vischer in “Volatility settles down as an Asset Class”, FT Mandate, calculated that volatility options strategies such as straddles and strangles were 10/20-times more expensive than volatility indexes in initiating volatility directional trades. Moreover, the Eurex volatility indexes of VSTOXX®, VDAX® and VSMI® allow for initiation of *leveraged relative value* implied volatility spread strategies. Diagrams 7a, 7b and 7c below look at the volatility spread history implied by the Eurex volatility index futures:

Diagram 7a: VSTOXX®/VDAX® Spread

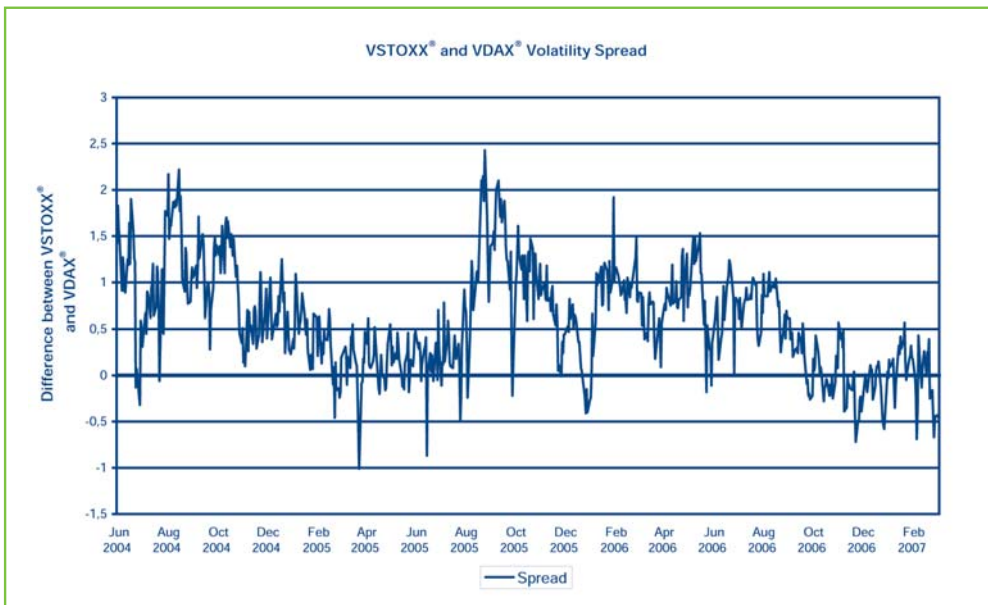


Diagram 7b: VDAX®/VSMI® Spread

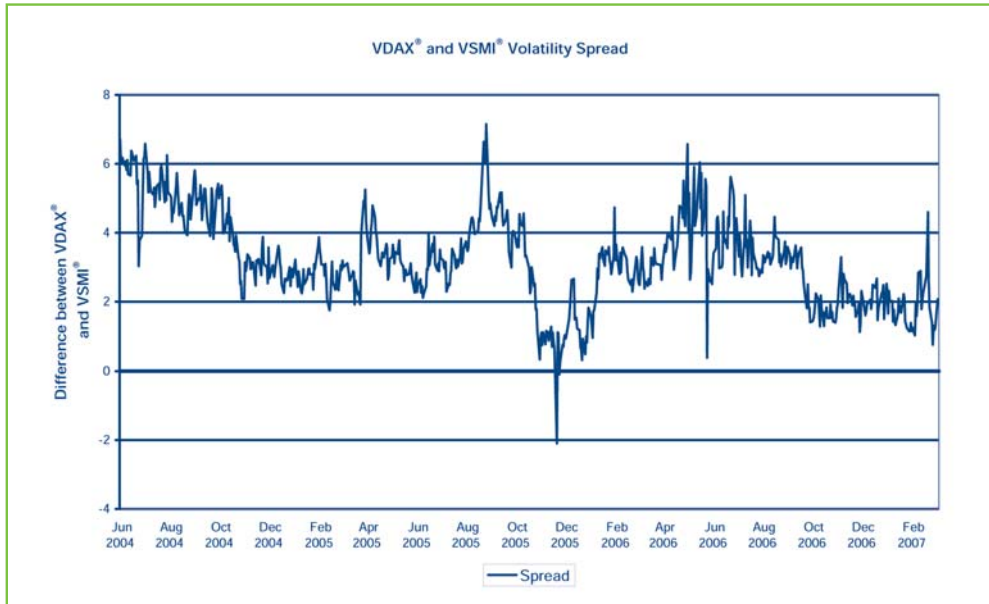
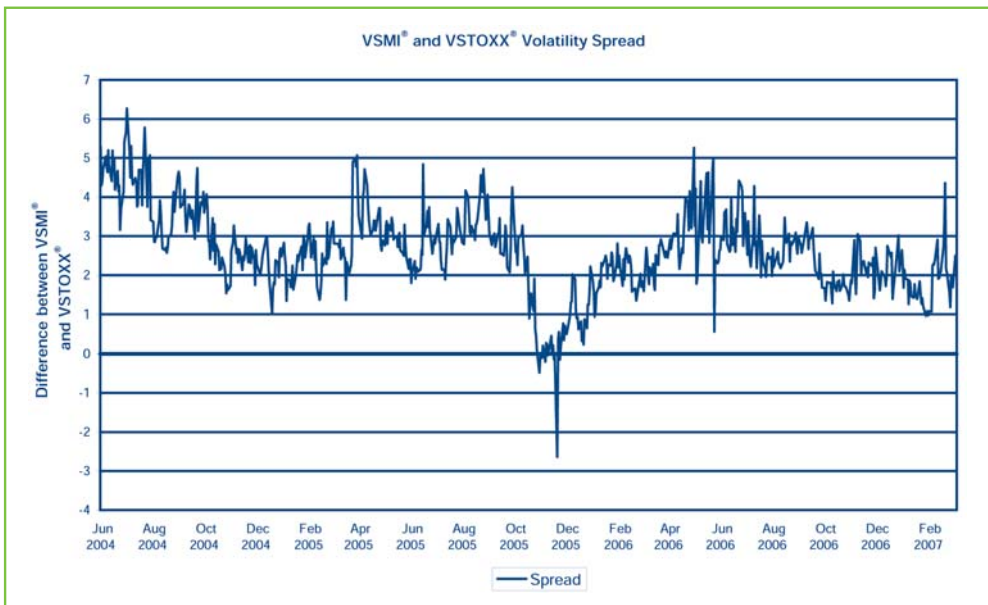


Diagram 7c: VSMI®/VSTOXX® Spread



Statistically (see appendix 3), the volatility spread changes have very little or no correlation⁸ with changes in the DAX®, SMI® and Dow Jones EURO STOXX 50® Equity Indexes and therefore offer an equity fund manager a source of alpha⁹ to transport to a benchmark equity portfolio based on those indexes.

How would a Eurex volatility index spread be structured? One method would be to structure the position in terms of the monetary value of a given change in volatility of each contract that is the monetary value

⁸ A. Cascon & W. Shadwick, "Alpha Purification & Omega Metrics", Eurex Xpand.

⁹ See B. Baldwin, "Is Portable Alpha Investing the answer for Pension Funds?" Pensions Week, December 2005.

of a 1% change in volatility. Obviously, based on this approach the ratio for the **VDAX®/VSTOXX®** spread would be **1:1** as a 1% change in volatility for both contracts is EUR 1,000. To structure a **VSTOXX®/VSMI®** volatility index spread, a 1% change in volatility for VSTOXX® (or VDAX®) is EUR 1,000, for VSMI® it is CHF 1,000 or currently at prevailing exchange rates, EUR 608.14 which gives a ratio of **1 VSTOXX® (or VDAX®):1.6 VSMI®**.

Eurex Block Trade Facility (BTF)

The **Eurex OTC Block Trade facility**¹⁰ (“BTF”) enables the initiation of relative value/cross asset class strategies, like those outlined above, in Eurex futures products off-exchange, whilst maintaining the benefits of having a position in exchange-traded derivative products, cleared by the Eurex clearing House – **independent daily mark-to-market valuation and substantially reduced counterparty risk**.

Diagram 8 below outlines the minimum amount of contracts that can be traded under the BTF:

Diagram 8: Eurex OTC Block Trade Facility

Contract	OTC Block Trade – Minimum Amount of Contracts
iTraxx® Credit Futures	2,500
iTraxx® Europe HiVol Index Futures	1,500
iTraxx® Europe Crossover Index Futures	1,000
Dow Jones EURO STOXX® Index Futures	1,000
Dow Jones EURO STOXX® Index Options	1,000
VStoxx® Volatility Index Futures	100
VDAX-New® Volatility Index Futures	100
VSMI® Volatility Index Futures	100

Conclusion

Because of their negative correlation to equity markets, the Eurex volatility index futures contracts are a very attractive asset class for the equity fund manager to increase portfolio diversification and potentially increase portfolio return. The contracts offer a very cheap (i.e. no transaction costs in managing delta) and leveraged (i.e. initial margin) way to buy and sell volatility and generate alpha returns by using a number of other Eurex futures products and initiating a variety of relative value/cross asset class strategies. Moreover, as exchange-traded derivative products, Eurex volatility index futures offer the *added benefit* over their OTC counterpart to fund management companies in terms of mark-to-market independent valuation and substantially reduced counterparty risk due to a central Clearing House.

¹⁰ See Eurex website: www.eurexchange.com/trading/market_model/wholesale/block_trades_en.html

References and Suggested Further Reading

See Eurex website for product information and specifications on the volatility index contracts:

www.eurexchange.com/trading/products/VOL_en.html

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H. Guo, Federal Reserve Bank of St. Louis Review, "Stock Market Returns, Volatility and Future Outputs", September/October 2004.

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J. Hill and S. Rattray, Goldman Sachs, "Volatility as a Tradable Asset", March 2004.

Appendix 1 – Eurex Volatility Futures Contract Specifications

Eurex Volatility Futures	
Underlying Instrument	Volatility indexes VSTOXX® (FVSX), VDAX-NEW® (FVDX) and VSMI® (FVSM)
Contract Value	EUR 1,000 per index point (FVSX, FVDX), CHF 1,000 per index point (FVSM)
Minimum Price Movement	0.05 of a point, equivalent to a value of EUR 50 and CHF 50 respectively
Last Trading Day	The Wednesday prior to the second last Friday of the expiration month (exactly 30 days before the next index option expiration)
Contract Months	The three nearest calendar months and the next quarterly month of the February, May, August, and November cycle
Daily Settlement	The closing price is determined within the closing auction. If no price can be determined in the closing auction or if the price determined does not reasonably reflect current market conditions, Daily Settlement Prices will be the last traded price within the last 15 minutes of continuous trading. If the last traded price is older than 15 minutes or does not reasonably reflect current market conditions, Eurex will establish the official settlement price.
Final Settlement	Cash settled
Final Settlement Price	Average over the index ticks of the last 30 minutes before expiration (FVSX: 11:30-12:00 CET, FVDX: 12:30-13:00 CET). Exception for FVSM, average over last 60 minutes: 09:00-10:00 CET
Trading Hours	09:00-17:30 CET

Appendix 2 – Calculation of Volatility Indexes

Firstly, the VDAX-NEW®, VSMI® and VSTOXX® subindexes are calculated using the eight Eurex option delivery expirations according to the formula shown below. The subindexes are calculated as the square root of implied variance taking into consideration the whole skew, both put and call wing, with a given weighting rather than just taking at-the-money volatility:

$$\text{VDAX-NEW}_i^\circ = 100 \times \sqrt{\sigma_i^2}, \text{VSMI}_i^\circ = 100 \times \sqrt{\sigma_i^2} \text{ and } \text{VSTOXX}_i^\circ = 100 \times \sqrt{\sigma_i^2}$$

Whereby:

$$\sigma_i^2 = \frac{2}{T_i} \sum_j \frac{\Delta K_{i,j}}{K_{i,j}^2} \times R_i \times M(K_{i,j}) - \frac{1}{T_i} \left(\frac{F_i}{K_{i,0}} - 1 \right)^2, i = 1, 2, \dots, 8$$

And:

T_i = Time to expiration of the i^{th} DAX®/SMI®/STOXX® option

F_i = Forward price derived from the prices of the i^{th} DAX®/SMI®/STOXX® option for which the absolute difference between call and put prices (C and P) is the smallest. Therefore:

$$F_i = K_{\min(C-P)} + R_i \times (C - P)$$

(Note: If a clear minimum does not exist, the average value of the relevant forward prices will be used instead)

$K_{i,j}$	= Exercise price of the j^{th} out-of-the-money option of the i^{th} DAX®/SMI®/STOXX® option expiration month both in ascending order.
$\Delta K_{i,j}$	= Interval between the relevant exercise prices or half the interval between the one higher and the one lower exercise price. On the boundaries, the simple interval between the highest and second highest exercise price (and lowest and second exercise price) is used:
$\Delta K_{i,j}$	= $\frac{K_{i,j+1} - K_{i,j-1}}{2}$
$K_{i,0}$	= Highest exercise price below forward price F_i
R_i	= Refinancing factor of the i^{th} DAX®/SMI®/STOXX® option
R_i	= $e^{r_i \times T_i}$
r_i	= Risk-free interest rate to the expiration of i^{th} DAX®/SMI®/STOXX® option
$M(K_{i,j})$	= Price of the option $K_{i,j}$, whereby $K_{i,j} \neq K_{i,0}$
$M(K_{i,0})$	= Average of the put and call prices at exercise price $K_{i,0}$

During the day, the respective best bid and best offer for all DAX®, SMI® and VSTOXX® option contracts listed on Eurex are taken at one minute intervals. The data is subject to filtering with all option prices that are one sided disregarded. Another filter verifies whether the options quoted are within the established maximum spreads. Mid option prices are then used.

Whilst the calculation of the volatility indexes take into consideration the option volatility skew, option prices below a minimum value of 0.5 are disregarded.

The VDAX-NEW®, VSMI®, VSTOXX® volatility indexes are then calculated based on a constant maturity of 30 days. They are derived by interpolation of the subindexes which are nearest to a remaining time to expiration of 30 days.

In terms of the VDAX-NEW® volatility index the interpolation calculation using the two subindexes nearest to 30 days would be:

$$\begin{aligned}
 \text{VDAX-NEW}^\circ &= 100 \times \sqrt{\left[T_i \times \sigma_i^2 \times \left[\frac{N_{T_{i+1}} - N_T}{N_{T_{i+1}} - N_{T_i}} \right] + T_{i+1} \times \sigma_{i+1}^2 \times \left[\frac{N_T - N_{T_i}}{N_{T_{i+1}} - N_{T_i}} \right] \right]} \times \frac{N_{365}}{N_T} \\
 &= \sqrt{\left[T_i \times \text{VDAX-NEW}_i^2 \times \left[\frac{N_{T_{i+1}} - N_T}{N_{T_{i+1}} - N_{T_i}} \right] + T_{i+1} \times \text{VDAX-NEW}_{i+1}^2 \times \left[\frac{N_T - N_{T_i}}{N_{T_{i+1}} - N_{T_i}} \right] \right]} \times \frac{N_{365}}{N_T}
 \end{aligned}$$

N_{T_i} = Time to expiration of the i^{th} Option series

$N_{T_{i+1}}$ = Time to expiration of the $i + 1^{th}$ Option series

N_T = Time for next x days

N_{365} = Time for a standard year

An Eurex volatility index future can be thought of as a forward on 30-day implied volatility on the VSTOXX®, VDAX-NEW® or VSMI® indexes. Carr and Wu in “A Tale of Two Indices” outlined an upper and lower boundary for the fair value of a volatility index future (their paper was based on the CBOE U.S. VIX® volatility index futures contract but it can obviously be applied to the pricing of the Eurex volatility index futures contracts).

Carr and Wu defined the upper and lower boundary of a volatility index future as:

$$E_0^Q \sqrt{RV_{T_1, T_2}} \leq F_0^{VOL} \leq \sqrt{E_0^Q RV_{T_1, T_2}}$$

Where the upper bound is the square root of the forward variance swap rate:

$$U_0 \equiv \sqrt{E_0^Q RV_{T_1, T_2}}$$

And the lower bound is the forward vol swap rate:

$$L_0 \equiv E_0^Q \sqrt{RV_{T_1, T_2}}$$

That is, the upper bound can be calculated via the 30-day implied volatility forward swap curve e.g. the VDAX-NEW® subindex and the lower bound by the 30-day implied volatility vola swap curve for example the ‘old’ ATM VDAX® subindex.

Based on these assumptions diagram 8 below looks at the upper and lower boundaries were calculated and compared to the actual VDAX® October 05 index future:

Diagram 8: Eurex Volatility Index Fair Value – Upper and Lower Boundary



Appendix 3 – Relationship Between Volatility Spreads and Stock Index Futures

Stock Index Futures	EURO STOXX®	DAX®	SMI®
Volatility Spreads			
VSTOXX®/VDAX®	0.0008/0.006	0.0003/0.001	N/A
VSMI®/VDAX®	N/A	-0.0067/0.25	-0.004/0.068
VSMI®/VSTOXX®	-0.012/0.31	N/A	-0.006/0.099

Period of Analysis: June 22, 2007-March 2, 7 2007.

First number in column is the correlation coefficient: the second number is the R2 'goodness of fit' statistic.



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