# EURO STOXX 50® Index Total Return Futures – Transition to €STR Flat

## Attachment 3 – Conversion Methodology

| | |
|---|---|
| Status | Final |
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| | |
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EURRX

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## 1. Introduction

In September 2018, the European Central Bank (ECB) working group on euro risk-free rates recommended to replace the euro overnight index average ($EONIA$) by the euro short-term rate ($€STR$) as the new euro risk-free rate.

It was also determined to recalibrate $EONIA$'s methodology to effectively become $€STR$ plus a fixed spread determined by the ECB. On 31 May 2019, ECB determined the one-off spread between $€STR$ and $EONIA$ as 0.085 % (i.e. 8.5 basis points) with the first $€STR$ fixing being published by the ECB on 2 October 2019.

The ECB working group on euro risk-free rates also recommends that trading participants gradually replace $EONIA$ with $€STR$ for all products and contracts, establishing $€STR$ as the standard reference rate.

In a first step and in line with ECB's recommendation, Eurex amended the Contract Specification of the EURO STOXX 50® Index Total Return Futures (TESX) on the transition on 2 October 2019, such that the referenced funding rate became $€STR$ plus the ECB provided spread of $8.5\ bp$, see [3].

In a second step, on 18 October 2021, Eurex will amend the Contract Specifications of TESX once more to rebase the reference funding rate on $€STR$ flat, i.e. removing ECB's legacy transitioning spread of $8.5\ bp$ going forward. The amendment of Contract Specification shall go along with a conversion approach, such that holders of open positions in TESX contracts are not economically impacted by the contractual change of the reference funding rate from $€STR + 8.5\ bp$ to $€STR$ flat.

The purpose of this paper is to describe the methodology of the conversion approach with respect to TESX contracts having Open Interest, i.e. open positions.

For detailed product information regarding the EURO STOXX 50® Index Total Return Futures (TESX), please refer to the Eurex website and the references therein, i.e. [1] and [2].

A formal interpretation of the product definition is presented in chapter 2 providing the necessary toolset employed in chapter 3 presenting the derivation of the conversion approach. An example calculation is provided as part of the appendix.

## 2. Formal Product Definition

This chapter provides a brief formal introduction of the Total Return Futures (TRF) as required for the derivation of the conversion methodology in chapter 3. Since focus will be put on the dependency of the TRF on the funding rate, the latter is explicitly stated as an input parameter to foster transparency.

As by the TRF product specification (cf. [2]), the daily settlement price in clearing notation in $t$ is given by

$$TRF_t(T,L) := S_t + Accrual_t(L) + Basis_t(T,L)$$

with past accrued dividend- less funding income

$$Accrual_t(L) := \sum_{\tau\in(\dots,t]} \left( \underbrace{Div_{t,k} 1_{\{\tau_k^{ex}=\tau\}}}_{\text{distributions}} - \underbrace{S_{\tau-1} L(\tau-1,\tau) \Delta_{SSP}(\tau-1,\tau)}_{\text{funding}} \right),$$

TRF basis

$$Basis_t(T,L) := S_t Y_t(T,L) \Delta_{SSP}(t,T),$$

day count fraction $\Delta_n(\cdot,\cdot)$ adjusted for $n$-day settlement period defined by

$$\Delta_n(t_0,t_1) := \frac{day(t_1,n,Cal) - day(t_0,n,Cal)}{DCB},$$

indicator function

$$1_{\{boolean\ expression\}} := \begin{cases} 1, & \text{if } boolean\ expression = \text{TRUE} \\ 0, & \text{else} \end{cases}$$

and

| | |
|---|---|
| $day(t,n,Cal)$ | Returns the day, $n$-business days after $t$ according to calendar $Cal$, the latter set to the calendar as specified in the product specification, i.e. $TARGET2$ |
| $DCB$ | Day Count Basis, i.e. 360 |
| $Div_{t,k}$ | Dividend payment of the $k$-th dividend ($k$ is employed to link the respective dividend amount with the relevant dates, i.e. cum-/ex-, record- and payment date) in $t$ according to the product specification |
| $L(t,t+1)$ | (Forward) funding rate according to the product specification fixed in $t$ and applicable to the period $t$ to $t+1$ |
| $SSP$ | Standard settlement period of the respective asset $S$, i.e. $SSP = 2$ business days |
| $S_t > 0$ | Closing price of the underlying index |
| $\tau_k^{ex}$ | Respective ex-date of the $k$-th dividend |
| $T$ | Maturity date of the respective series of the product |
| $Y_t(T,L)$ | So-called TRF spread in $t$ with respect to TRF maturity date $T$ and (forward) funding rate $L$ |

The TRF spread $Y_t(\cdot,\cdot)$ can inversely be inferred from the TRF basis by

$$Y_t(T,L) = \begin{cases} \dfrac{Basis_t(T,L)}{S_t \Delta_{SSP}(t,T)}, & \text{if } t < T \wedge S_t \neq 0 \\ 0, & \text{else} \end{cases}.$$

The TRF trading notation corresponds to $Y_t(\cdot,\cdot)$[1], the TRF clearing notation corresponds to $TRF_t(\cdot,\cdot)$ and constitutes the basis for subsequent variation margins.

The clearing notation can also be expressed in terms of the TRF spread by

$$TRF_t\big(T, Y_t(T,L)\big) = Accrual_t + S_t\big(1 + Y_t(T,L)\Delta_{SSP}(t,T)\big).$$

The final settlement price of the TRF is determined as

$$TRF_T(T,L) = S_T + Accrual_T(L),$$

exhibiting a vanishing TRF basis as time approaches expiry. With the day count fraction becoming zero on expiry, the notion of the TRF spread also vanishes.

[1] For notational simplicity, the conversion into basis points is omitted here. The actual figure traded in the orderbook is the basis point representation of $Y_t(\cdot,\cdot)$.

## 3. Derivation of Conversion Methodology

The derivation of the conversion adjustment adopts the general approach of considering the spread of $8.5\ bp$ flat along the forward term structure.

For the derivation of the conversion methodology a clear differentiation between historic and forward funding on one hand and between two different forward funding regimes on the other hand is necessary, therefore the following notation is introduced:

Any contractual historical funding is referred to by $H(\cdot,\cdot)$ and given by

$$H(\tau-1,\tau) := L(\tau-1,\tau) \text{ for } \tau \leq t.$$

Any contractual forward funding, i.e. the part of the funding that is still to be realized, is referred to by $G(\cdot,\cdot)$ and given by

$$G(\tau-1,\tau) := L(\tau-1,\tau) \text{ for } \tau > t.$$

To further clearly differentiate between the two different forward funding regimes, $G^+$ is introduced for $€STR + 8.5\ bp$ and $G$ for $€STR$ flat. This yields the following relation $G^+ = G + 8.5\ bp$, which will be employed throughout this section.

As is apparent from the revised notation, the historically accrued component $Accrual_t(H)$ of distributions less funding is not affected by the switch in (forward) funding from $G^+$ to $G$ and will remain unchanged.

Thus, the overall goal of this section is to derive the difference between the switch induced change in TRF spread from $Y_t(T,G^+)$ to $Y_t(T,G)$, respectively the change in clearing notation from $TRF_t\big(T,Y_t(T,G^+)\big)$ to $TRF_t\big(T,Y_t(T,G)\big)$. In order to adhere to trading conventions, results will be derived based on the TRF spread, rounded to multiples of $0.5\ bp$.

### 3.1. Assumptions

The following assumptions are made to support the subsequent derivation of the conversion methodology:

1) The switch of TESX inherent (forward) funding from $G^+$ to $G$, i.e. from $€STR + 8.5\ pb$ to $€STR$ flat, has no impact on the valuation of unrelated derivatives having EURO STOXX 50® Index (SX5E) as underlying such as options and/or forwards/futures. This is meaningful, since their valuation does not rely on any explicit funding assumption but is rather determined as the market's equilibrium funding expressed via market prices.

   This assumption implies that there is also no change in the genuine market implied funding/lending/repo rate nor dividend expectations regarding unrelated derivatives having SX5E as underlying.

2) Based on a forward term structure of SX5E with tenor points aligned to TESX expiries, as provided by Eurex and described in [4], linear interpolation is suitable to obtain any missing tenor points. The provided forward term structure is referenced by $fwd_t(T_i)$ and likewise denoted as $fwd_t(\tau)$ for any $\tau \geq t$.

   This assumption implies a viable evaluation of the conversion spread based on public data and without having to rely on model assumptions regarding valuation and/or valuation data.

### 3.2. TRF Conversion Settlement Spread

Evaluating the TRF in $t$ by taking today's expectation of the final payoff in $T$ provides

$$\begin{aligned} TRF_t(T,G^+) &= E_t[TRF_T(T,G^+)] \\ &= E_t[S_T + Accrual_T]. \end{aligned}$$

By comparison with the product definition this yields the following view on the TRF basis

$$\begin{aligned} Basis_t(T,G^+) &= E_t[S_T + Accrual_T] - (S_t + Accrual_t) \\ &= \underbrace{E_t[S_T] - S_t + E_t\left[\sum_{\tau\in(t,T]} Div_{t,k} 1_{\{\tau_k^{ex}=\tau\}}\right]}_{c:=} - \underbrace{E_t\left[\sum_{\tau\in(t,T]} S_{\tau-1} G^+(\tau-1,\tau)\Delta_{SSP}(\tau-1,\tau)\right]}_{\text{future expected TRF funding}}. \end{aligned}$$

As by assumption 1), the first element is not impacted by the switch in funding – contrary to the second element. By definition and for the non-trivial case, the TRF basis translates into the TRF spread as

$$Y_t(T,G^+) = \frac{Basis_t(T,G^+)}{S_t\Delta_{SSP}(t,T)} = \frac{c - E_t\left[\sum_{\tau\in(t,T]} S_{\tau-1} G^+(\tau-1,\tau)\Delta_{SSP}(\tau-1,\tau)\right]}{S_t\Delta_{SSP}(t,T)}.$$

Decomposition of $G^+$ into $G + 8.5\ bp$ along the whole term structure yields

$$Y_t(T,G^+) = \underbrace{\frac{c - E_t\left[\sum_{\tau\in(t,T]} S_{\tau-1} G(\tau-1,\tau)\Delta_{SSP}(\tau-1,\tau)\right]}{S_t\Delta_{SSP}(t,T)}}_{Y_t(T,G)=} - \frac{8.5\ bp \cdot E_t\left[\sum_{\tau\in(t,T]} S_{\tau-1}\Delta_{SSP}(\tau-1,\tau)\right]}{S_t\Delta_{SSP}(t,T)}.$$

The first term can be identified as the genuine Conversion Settlement Spread $Y_t(T,G)$. Thus, it remains to obtain a suitable estimation of the second term, or rather the expectation therein, given the remainder is deterministic.

Applying assumption 2) yields

$$E_t\left[\sum_{\tau\in(t,T]} S_{\tau-1}\Delta_{SSP}(\tau-1,\tau)\right] = \sum_{\tau\in(t,T]} E_t[S_{\tau-1}]\Delta_{SSP}(\tau-1,\tau) \cong \sum_{\tau\in(t,T]} fwd_t(\tau-1)\Delta_{SSP}(\tau-1,\tau),$$

which allows to obtain a reasonably accurate approximation of the spread, which is still straight forward to calculate, while not having to rely on any modelling assumptions, by

$$Y_t(T,G^+) \cong Y_t(T,G) - \frac{8.5\ bp \cdot \sum_{\tau\in(t,T]} fwd_t(\tau-1)\Delta_{SSP}(\tau-1,\tau)}{S_t\Delta_{SSP}(t,T)}.$$

The approximation of the Conversion Settlement Spread $Y_t(T,G)$ is simply given by rearranging

$$Y_t(T,G) \cong Y_t(T,G^+) + \frac{8.5\ bp \cdot \sum_{\tau\in(t,T]} fwd_t(\tau-1)\Delta_{SSP}(\tau-1,\tau)}{S_t\Delta_{SSP}(t,T)}.$$

Note, the TRF Conversion Settlement Spread $Y_t(T,G)$ is higher than the TRF daily settlement spread $Y_t(T,G^+)$ observed before the switch in funding. This is an economically meaningful result, since the future funding cost expressed via the spread decrease ceteris paribus.

### 3.3. TRF Conversion Settlement Price

The TRF Conversion Settlement Price is simply given by the conversion of the TRF Conversion Settlement Spread into clearing notation, i.e. by $TRF_t\big(T, Y_t(T,G)\big)$.

Since the TRF price is linear in the TRF spread, an increase in the TRF conversion spread directly implies and increase in the TRF conversion price ceteris paribus. This is an immediate consequence of the change in TRF Spread and noting that the conversion from TRF spread into TRF basis is nothing but a conversion from a rate into a monetary figure.

### 3.4. Conversion via Technical Trades

Any holder of an open position in a TESX contract shall be compensated for the amendment of the Contract Specifications, i.e. the switch in (forward) funding. Since the latter results in elevated TRF prices from a valuation point of view, as derived in 3.2 and 3.3, Eurex Clearing will process technical trades that re-establish open positions at compensation settlement prices. Thereby reference prices for determining variation margins on the next trading day are adjusted to effectively compensate for the funding switch. A summary of the operational process is given below.

At the end of day $t$, 15 October 2021, Eurex Clearing will:

1) Calculate the Daily Settlement Price $TRF_t\big(T_i, Y_t(T_i, G^+)\big)$ for all expiries $T_i$ and determine all margins payments as usual.
2) Calculate the Conversion Settlement Price $TRF_t\big(T_i, Y_t(T_i, G)\big)$ for each expiry $T_i$, based upon the derivations above, employing the Conversion Settlement Spread $Y_t(T_i, G)$, rounded to multiples of 0.5 basis points and otherwise unchanged values of Index Close $S_t$, Accrued Distributions and Accrued Funding $Accrual_t(H)$ as employed for the calculation of the Daily Settlement Price in 1).
3) Re-establish all open positions via dedicated technical trades, prior to the start of trading on the next trading day (18 October 2021). More specifically all open positions are closed-out at previous day's Daily Settlement Prices $TRF_t\big(T_i, Y_t(T_i, G^+)\big)$ and re-opened at Conversion Settlement Prices $TRF_t\big(T_i, Y_t(T_i, G)\big)$, ceteris paribus.

## 4. References

[1] Eurex Website – Total Return Futures (TRF)[2]

[2] Contract Specifications for Futures Contracts and Options Contracts at Eurex Deutschland[3]

[3] Eurex Circular 087/19: EURO STOXX 50® Total Return Futures and EURO STOXX 50® Index Variance Futures: Amendments related to the introduction of the euro short-term rate (€STR)[4]

[4] Eurex, "EURO STOXX 50® Index Total Return Futures – Transition to €STR Flat. Attachment 2 – Determination of EURO STOXX 50® Index (SX5E) forward points for use in the conversion methodology", 2021.

[2] www.eurex.com/ex-en/markets/idx/trf

[3] www.eurex.com/ex-en/rules-regs/eurex-rules-regulations/Contract-Specifications-53644

[4] www.eurex.com/ex-en/find/circulars/circular-1612326

## A. Appendix

### A.1. Example Calculation of Conversion Settlement Prices

Assuming the conversion approach would have been applied on sample date of 2020/09/18, the following Conversion Settlement Prices would have been obtained.

The following data fed into daily settlement prices of TESX on the sample date. Values are rounded (if applicable) for ease of depiction.

| | | |
|---|---|---|
| Past Distributions | $Accrual_t(H)$ | 490.96 |
| Index Close | $S_t$ | 3283.69 |

| | | Daily Settlement Figures | |
|---|---|---|---|
| Contract | Expiry Date $T_i$ | Spread $Y_t(T_i, G^+)$ | Price $TRF_t(T_i, Y_t(T_i, G^+))$ |
| TESX DEC20 | 2020-12-18 | -6.5 | 3774.11 |
| TESX MAR21 | 2021-03-19 | -0.5 | 3774.57 |
| TESX JUN21 | 2021-06-18 | 25.0 | 3780.88 |
| TESX SEP21 | 2021-09-17 | 21.0 | 3781.63 |
| TESX DEC21 | 2021-12-17 | 23.0 | 3784.20 |
| TESX MAR22 | 2022-03-18 | 26.5 | 3787.85 |
| TESX JUN22 | 2022-06-17 | 34.5 | 3794.70 |
| TESX SEP22 | 2022-09-16 | 32.5 | 3796.23 |
| TESX DEC22 | 2022-12-16 | 35.0 | 3800.80 |
| TESX MAR23 | 2023-03-17 | 41.0 | 3808.68 |
| TESX JUN23 | 2023-06-16 | 46.0 | 3816.65 |
| TESX SEP23 | 2023-09-15 | 47.0 | 3821.47 |
| TESX DEC23 | 2023-12-15 | 46.0 | 3824.29 |
| TESX MAR24 | 2024-03-15 | 50.0 | 3832.76 |
| TESX JUN24 | 2024-06-21 | 54.0 | 3842.23 |
| TESX SEP24 | 2024-09-20 | 51.0 | 3842.71 |
| TESX DEC24 | 2024-12-20 | 55.0 | 3852.61 |
| TESX MAR25 | 2025-03-21 | 60.0 | 3864.68 |
| TESX JUN25 | 2025-06-20 | 59.0 | 3868.08 |
| TESX SEP25 | 2025-09-19 | 61.0 | 3876.31 |
| TESX DEC25 | 2025-12-19 | 63.0 | 3884.87 |
| TESX DEC26 | 2026-12-18 | 70.0 | 3920.36 |
| TESX DEC27 | 2027-12-17 | 79.5 | 3966.53 |
| TESX DEC28 | 2028-12-15 | 83.5 | 4003.90 |
| TESX DEC29 | 2029-12-21 | 90.5 | 4053.91 |

The following implied index forwards were obtained for SX5E[5].

| Contract | Expiry Date $T_i$ | Implied Index Forward $fwd_t(T_i)$ |
|---|---|---|
| SEP20 | 2020-09-18 | 3283.69 |
| OCT20 | 2020-10-16 | 3287.23 |
| NOV20 | 2020-11-20 | 3283.20 |
| DEC20 | 2020-12-18 | 3280.00 |
| JAN21 | 2021-01-15 | 3275.77 |
| FEB21 | 2021-02-19 | 3269.36 |
| MAR21 | 2021-03-19 | 3266.58 |
| JUN21 | 2021-06-18 | 3215.75 |
| SEP21 | 2021-09-17 | 3202.00 |
| DEC21 | 2021-12-17 | 3188.78 |
| MAR22 | 2022-03-18 | 3177.12 |
| JUN22 | 2022-06-17 | 3124.19 |
| SEP22 | 2022-09-16 | 3112.92 |
| DEC22 | 2022-12-16 | 3097.77 |
| JUN23 | 2023-06-16 | 3032.22 |
| DEC23 | 2023-12-15 | 3011.93 |
| DEC24 | 2024-12-20 | 2937.22 |
| DEC25 | 2025-12-19 | 2868.04 |
| DEC26 | 2026-12-18 | 2800.06 |
| DEC27 | 2027-12-17 | 2735.21 |
| DEC28 | 2028-12-15 | 2671.48 |
| DEC29 | 2029-12-21 | 2610.74 |

[5] The derivation of the forward curve was simplified for the example stated here by a straightforward put/call-parity regression on the basis of daily OESX settlement prices.

Combining the results from above along with a suitable calendar, allows the computation of Conversion Settlement Spreads as well as Conversion Settlement Prices.

| | | Conversion Settlement Figures | |
|---|---|---|---|
| Contract | Expiry Date $T_i$ | Spread $Y_t(T_i, G)$ | Price $TRF_t\big(T_i, Y_t(T_i, G)\big)$ |
| TESX DEC20 | 2020-12-18 | 2.0 | 3774.82 |
| TESX MAR21 | 2021-03-19 | 8.0 | 3775.98 |
| TESX JUN21 | 2021-06-18 | 33.5 | 3782.99 |
| TESX SEP21 | 2021-09-17 | 29.5 | 3784.45 |
| TESX DEC21 | 2021-12-17 | 31.5 | 3787.73 |
| TESX MAR22 | 2022-03-18 | 35.0 | 3792.08 |
| TESX JUN22 | 2022-06-17 | 43.0 | 3799.64 |
| TESX SEP22 | 2022-09-16 | 41.0 | 3801.88 |
| TESX DEC22 | 2022-12-16 | 43.5 | 3807.15 |
| TESX MAR23 | 2023-03-17 | 49.0 | 3815.33 |
| TESX JUN23 | 2023-06-16 | 54.0 | 3823.96 |
| TESX SEP23 | 2023-09-15 | 55.0 | 3829.44 |
| TESX DEC23 | 2023-12-15 | 54.0 | 3832.92 |
| TESX MAR24 | 2024-03-15 | 58.0 | 3842.05 |
| TESX JUN24 | 2024-06-21 | 62.0 | 3852.24 |
| TESX SEP24 | 2024-09-20 | 59.0 | 3853.39 |
| TESX DEC24 | 2024-12-20 | 63.0 | 3863.95 |
| TESX MAR25 | 2025-03-21 | 68.0 | 3876.68 |
| TESX JUN25 | 2025-06-20 | 67.0 | 3880.75 |
| TESX SEP25 | 2025-09-19 | 69.0 | 3889.64 |
| TESX DEC25 | 2025-12-19 | 71.0 | 3898.87 |
| TESX DEC26 | 2026-12-18 | 78.0 | 3937.01 |
| TESX DEC27 | 2027-12-17 | 87.5 | 3985.84 |
| TESX DEC28 | 2028-12-15 | 91.0 | 4024.50 |
| TESX DEC29 | 2029-12-21 | 98.0 | 4077.06 |

Figure 2 and Figure 1 depict the results from above and conclude the example.

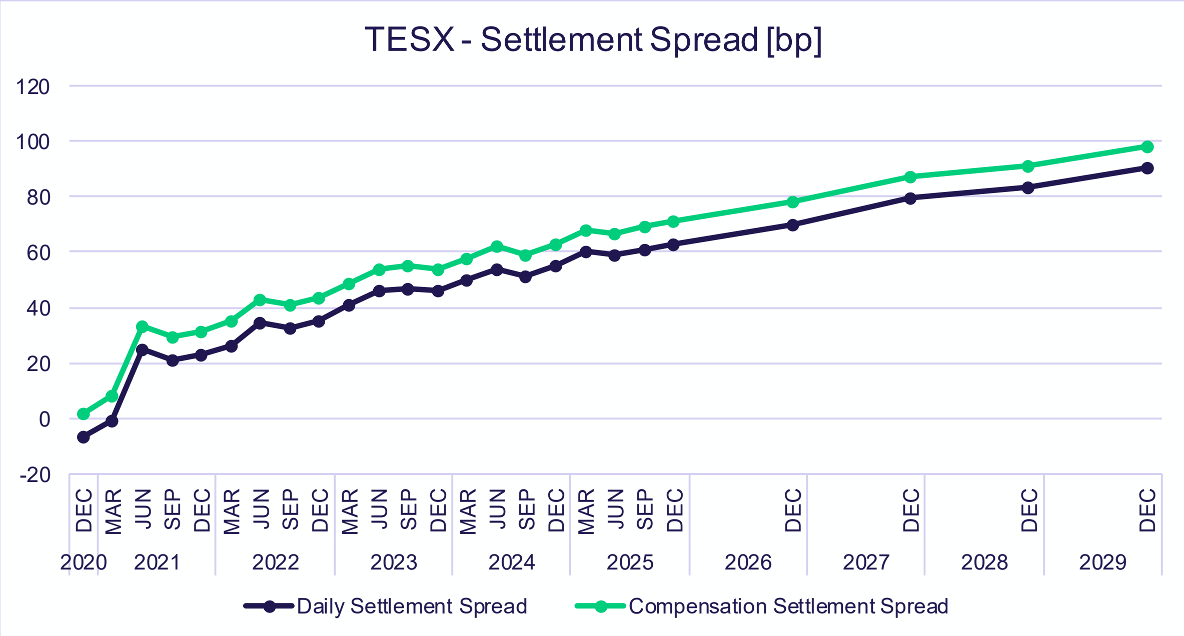


Figure 2 Comparison of Daily- and Conversion Settlement Spread

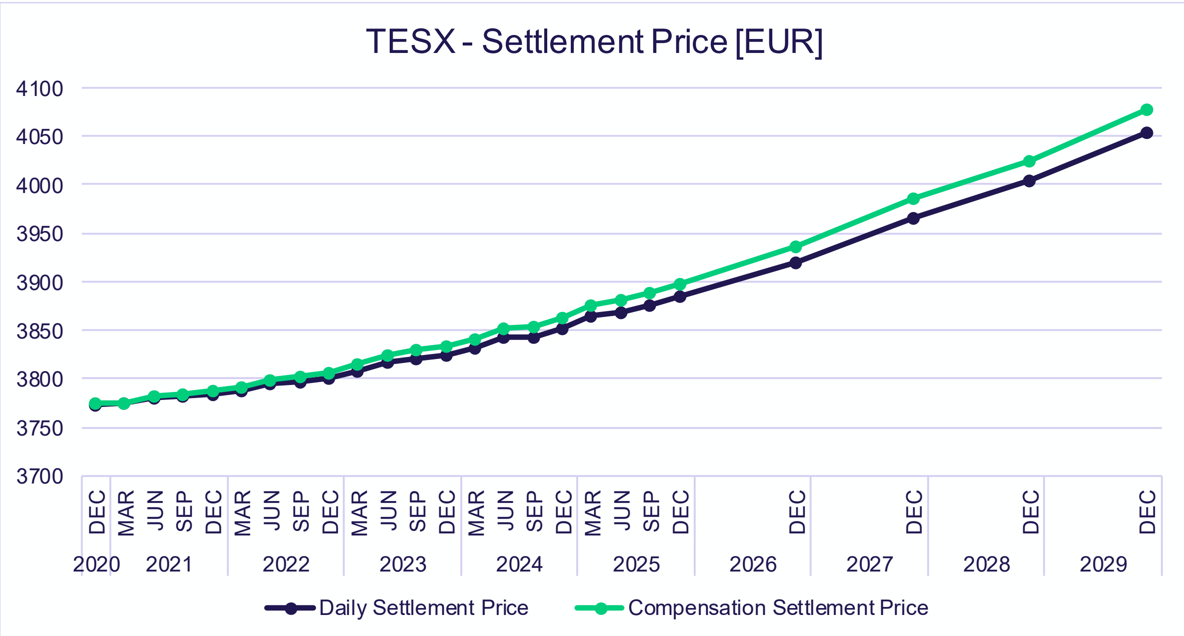


Figure 1 Comparison of Daily- and Conversion Settlement Price